Marks

[15] **1.** Suppose that the electrical potential V in space is given by the function

$$V(x, y, z) = x^2 + 2y^2 + e^z.$$

(a) What is the equation of the tangent plane to the equipotential surface $V(x, y, z) = 3$ at $x = 1$, $y = 1$, $z = 0$.

(b) A particle moves along the path $\mathbf{r}(t) = \langle t, t^2, t^3 - 1\rangle$. What is the rate of change of the electrical potential when the particle passes through the point $\langle 1, 1, 0\rangle$.

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[14] **2.** Find the absolute minimum of the function $f(x, y) = 4 + 2xy - x - y$ on the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 2$.

Continued on page 4

[14] **3.** For what value of a is the vector field

$$\mathbf{F} = \langle axe^{2y+z}, 2x^2e^{2y+z}, x^2e^{2y+z} \rangle$$

conservative? For this value of a find a function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

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[12] **4.** Let E be the solid (in the first octant) bounded by the coordinate planes and two parabolic cylinders $z = 1 - x^2$ and $z = 1 - y^2$. Make a sketch of E and find its volume.

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[10] **5.** Evaluate the iterated integral

$$\int_{-1}^{1} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{0}^{x^2+z^2} y\, dy\, dx\, dz.$$

Make a sketch of the region of integration.

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[15] **6.** Let S be the surface of the paraboloid

$$z = 9 - x^2 - y^2$$

that remains above the xy plane (i.e., $z \geq 0$) oriented with an upward normal.

(a) What is the boundary curve $C = \partial S$ and what direction is its positive orientation?

(b) What surface S_1 in the xy plane, with what assignment of normal, has the same boundary curve as S with the same orientation?

(c) Evaluate $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = \langle xe^z - 3y, ye^{z^2} + 2x, x^2y^2z^2 \rangle$$

(We are using the notation $\nabla \times \mathbf{F}$ for **curl** $(\mathbf{F})$.)

Continued on page 8

[20] **7.** Consider the vector field

$$\mathbf{F}(x, y, z) = (x^2 + y^2 + z^2)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

(a) Evaluate $\nabla \times \mathbf{F}$ and $\nabla \cdot \mathbf{F}$. (We are using the notation $\nabla \times \mathbf{F}$ for **curl** ($\mathbf{F}$) and $\nabla \cdot \mathbf{F}$ for **div** ($\mathbf{F}$)).

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve witn parametrization

$$\mathbf{f}(t) = \langle 2\sin t, 3\cos t, 3\rangle, \quad 0 \le t \le 2\pi.$$

Continued on page 9

(c) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the surface of the solid bounded by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the plane $z = 0$, with outward pointing normal vector.

The End

Be sure that this examination has 9 pages including this cover

The University of British Columbia

Sessional Examinations - December 2006

Mathematics 263

Multivariable and Vector Calculus

Closed book examination — Time: 2.5 hours

Print Name ________________ **Signature** ________________

Student Number ________________ **Instructor's Name** ________________

Section Number ________________

Special Instructions:

No calculators, cell phones, or books are allowed.
You may bring one letter-sized formula sheet.
For all questions, you must show your work (i.e., intermediate steps) for full credit.

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		15
2		14
3		14
4		12
5		10
6		15
7		20
Total		100