The University of British Columbia

Final Examination - April, 2007

Mathematics 257/316

Closed book examination Time: 2.5 hours

Instructor Name: ____________________

Last Name: ____________________, **First:** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

- Be sure that this examination has 9 pages. Write your name on top of each page.
- A standard size (both sides) sheet of notes is allowed in this examination.
- Calculators are not allowed in this examination.
- Answers must be justified to receive full credit.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		15
2		10
3		25
4		35
5		15
Total		100

[15] **1**. a) Find the Fourier sine series of the function

$$g(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 0 & \text{if } 1 \leq x \leq 2 \end{cases}$$

b) Sketch the graph of the function to which the series converges over at least two periods. On your graph, specify the points x where Gibbs phenomenon occurs.

[10] **2**. Consider an elastic string of length L whose ends are held fixed, and with a given a^2. The string is set in motion from its equilibrium position with an initial velocity $g(x)$.

a) Write the initial boundary value problem satisfied by the displacement $u(x,t)$ of the string.

b) Assume now that $L = 2$ and $a = 1$. Find $u(x,t)$ when $g(x)$ is the function given in Problem 1:

$$g(x) = \begin{cases} x & \text{if } 0 \le x < 1 \\ 0 & \text{if } 1 \le x \le 2 \end{cases}$$

[25] **3**. Find the general solution of the following boundary value problem in the semi-infinite strip $\{(x,y)\,;\, 0<x<1,\ y>0\}$:

$$\begin{cases} \Delta u = 0, & \text{for all } 0<x<1,\ y>0 \\ u_x(0,y)=0, \quad u_x(1,y)=0 & \text{for all } y>0 \\ u(x,0)=f(x), \quad u(x,y) \text{ bounded as } y \text{ approaches } \infty \text{ for all } 0<x<1 \end{cases}$$

(use separation of variables).

[35] **4**. a) Determine the form of the eigenfunctions and the equation satisfied by the eigenvalues of the following Sturm-Liouville problem:

$$y'' + \lambda y = 0, \qquad 0 < x < 1$$

$$y(0) = 0, \quad y(1) + 2\,y'(1) = 0$$

Show that there exists an infinite sequence λ_n of eigenvalues and estimate λ_n for large values of n.

b) Express the function

$$f(x) = x, \qquad 0 < x < 1$$

as a series of eigenfunctions of this Sturm-Liouville problem (you may normalize the eigenfunctions first).

c) Find the solution of the following non-homogeneous initial boundary value problem (you may use your answers to parts (a) and (b)):

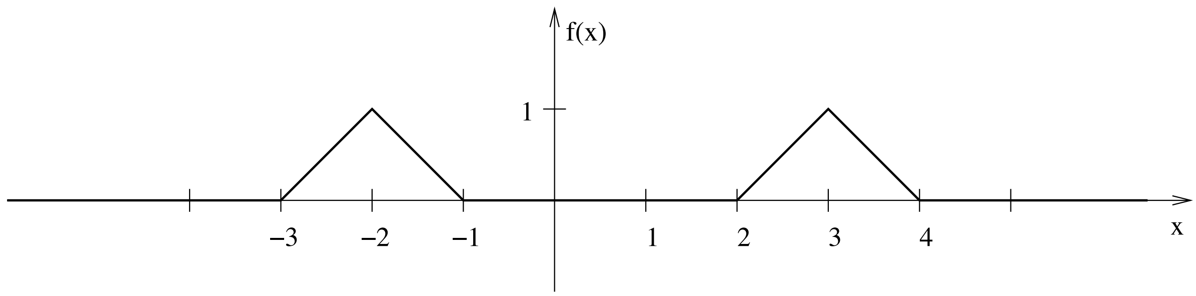
$$\begin{cases} u_t = u_{xx} + xt \qquad 0 < x < 1 \quad t > 0 \\ u(0,t) = 0 \qquad t > 0 \\ u(1,t) + 2\,u_x(1,t) = 0 \qquad t > 0 \\ u(x,0) = 0 \qquad 0 < x < 1 \end{cases}$$

[15] **5**. Consider the wave equation $u_{tt} = u_{xx}$ for an infinite string (x in $(-\infty, \infty)$) with initial conditions

$$u(x,0) = f(x), \qquad u_t(x,0) = g(x).$$

a) Write d'Alembert's formula for the solution $u(x,t)$.

b) Sketch the solution for $t = 1$ and $t = 2$, and explain its behaviour as t increases when $g(x) = 0$ and $f(x)$ is given by the following graph:



c) Sketch the solution for $t = 0$, $t = 1$ and $t = 2$ when

$$f(x) = 0, \qquad g(x) = \begin{cases} 1 & \text{if } -6 < x < -2, \\ 1 & \text{if } 2 < x < 4, \\ 0 & \text{otherwise.} \end{cases}$$