

Marks

[20] 1. Linear differential equations.

(a) Solve the initial value problem:

$$y' + 3y = 2e^{-2t}, \quad y(0) = 1.$$

(b) Solve the initial value problem:

$$y' + 2ty = e^{-t^2}, \quad y(0) = 3.$$

(c) Solve the initial value problem:

$$y'' + 2y' + y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

(d) Solve the initial value problem:

$$y'' + 2y' + y = 2e^{-t}, \quad y(0) = 0, \quad y'(0) = 0,$$

and show that the solution  $y(t)$  has maximum value of  $4e^{-2}$ . At what time is this value attained?

Marks

- [20] **2.** Find the general solution of the following homogeneous linear system:

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} \mathbf{x},$$

and sketch the phase plane close to  $\mathbf{x} = 0$ .

(b) Find the solution of the following initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 12t - 11 \\ -3 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

[20] **3.** The function  $f(t)$  is defined for  $t \in [-1, 1]$  by

$$f(t) = \begin{cases} -t - 1, & t \in [0, 1], \\ 1 - t, & t \in [0, 1], \end{cases}$$

- (a) Sketch the function  $f(t)$  over the interval  $t \in [-1, 1]$ , and find the Fourier series for  $f(t)$ . Is the function odd or even?
- (b) To what values should the Fourier series of part (a) converge to at: (i)  $t = 0.5$ ; (ii)  $t = 2$ ; (iii)  $t = 3$ ; (iv)  $t = 3.5$ ?

- (c) Find the general solution of:

$$y'' + \omega^2 y = f(t),$$

where  $f(t)$  is the function in part (a), assuming  $\omega$  is a positive constant satisfying  $\omega < \pi$ .

- (d) If instead we have  $\omega \geq \pi$ , for what value(s) of  $\omega$  will your solution become very large as  $t \rightarrow \infty$ ?

- [20] 4. Consider the the following IBVP

$$u_t = u_{xx} - \cos 3\pi x, \quad 0 < x < 1, \quad t \geq 0,$$

subject to the boundary and initial conditions:

$$u_x(0, t) = 2, \quad u_x(1, t) = 2, \quad u(x, 0) = 2x + \cos \pi x$$

- (a) Writing  $u(x, t) = u_s(x) + v(x, t)$  state the problems that are satisfied by the steady state solution,  $u_s(x)$ , and the transient solution  $v(x, t)$ .
- (b) Find the steady state solution,  $u_s(x)$ .

- (c) Find the transient solution,  $v(x, t)$  using separation of variables, and hence write down the full solution,  $u(x, t)$ .
- (d) Show that

$$u(0.25, t) \rightarrow \frac{1}{2} + \frac{1}{\sqrt{2}}, \text{ as } t \rightarrow \infty,$$

and estimate how large  $t$  must be in order for

$$\left| u(0.25, t) - \frac{1}{2} - \frac{1}{\sqrt{2}} \right| < 0.01.$$

- [20] 5. Use separation of variables to solve the BVP:

$$u_{xx} + u_{yy} = 0, \quad 0 < x < 1, \quad 0 < y < 1,$$

subject to the boundary conditions:  $u(0, y) = 1$ ,  $u_x(1, y) = 0$ ,  $u(x, 0) = 0$ ,  $u(x, 1) = \sin 5\pi x$ .



Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - April 2009

Mathematics 256

Differential Equations

Closed book examination

Time:  $2\frac{1}{2}$  hours

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

**Special Instructions:**

Two  $8\frac{1}{2}$ "  $\times$  11" two sided cheat sheets are permitted.

Non-programmable calculators allowed.

Show your work in the spaces provided.

You are encouraged to explain all steps in your solutions.

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1. Each candidate must be prepared to produce, upon request, a library/AMS card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
  - (b) Speaking or communicating with other candidates.
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5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1		20
2		20
3		20
4		20
5		20
Total		100