Final Exam
Math 253
Dec 4th, 2013

Last Name: ____________ First Name: ____________

Student # : ____________ Instructor's Name : ____________

Instructions:
No memory aids allowed. No calculators allowed. No communication devices allowed. Use the space provided on the exam. If you use the back of a page, write "see back" on the front of the page. This exam is 180 minutes long.

Question	Points	Score
1	21	
2	12	
3	6	
4	8	
5	9	
6	12	
7	12	
8	20	
Total:	100	

1. The following questions will be graded by answer only.

   (a) $\boxed{\text{3 points}}$ Find a unit vector, with a positive **k** component, which is parallel to the plane $x - 2y + z = 3$ and perpendicular to the vector $\langle 1, 1, 1 \rangle$.

   (b) $\boxed{\text{3 points}}$ Let $z = \frac{1}{3}(1 + xy^2)^3$, $x = g(t)$, and $y = h(t)$. Suppose that $g(0) = 2$, $h(0) = 1$, $g'(0) = -3$, and $h'(0) = 5$. Compute the value of $\frac{dz}{dt}$ when $t = 0$.

   (c) $\boxed{\text{6 points}}$ Let $z(x, y)$ be defined implicitly by the equation $z^3 + z + x + y^2 = 3$. Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ and $\frac{\partial^2 z}{\partial x \partial y}$.

(d) $\boxed{\text{3 points}}$ Find the area of the triangle with vertices $(1, 2, 3)$, $(4, 6, 2)$, $(2, 4, 3)$.

(e) $\boxed{\text{3 points}}$ Let $u(x, t) = e^{t+ax} + e^{t-ax}$ where a is a parameter. Find a such that $5u_t = u_{xx} + u$.

(f) $\boxed{\text{3 points}}$ A line through the origin makes an angle of 60 degrees with the x-axis and with the y-axis. What angle does it make with the z-axis?

2. The temperature is given by the function $T(x,y,z) = x^3 + 5yz^2 - 17z$.

   (a) $\boxed{\text{3 points}}$ In what direction (given by a unit vector) does the temperature **decrease** fastest at the point $(-1,2,1)$?

   (b) $\boxed{\text{3 points}}$ If you are at $(-1,2,1)$ does the temperature increase faster if you walk towards the point $(3,2,1)$ or towards the point $(-1,3,2)$? (show all your work!)

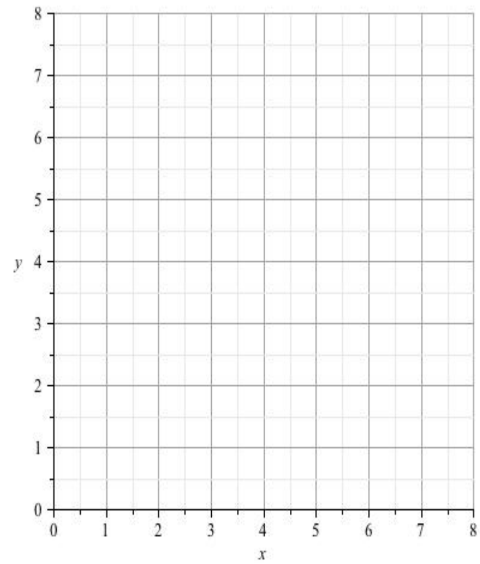
   (c) $\boxed{\text{3 points}}$ Find the tangent plane to the level surface of T at the point $(-1,2,1)$.

   (d) $\boxed{\text{3 points}}$ Using the value of T at $(-1,2,1)$ estimate the temperature at the point $(-0.98, 2.01, 0.97)$.

3. Consider the integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 \frac{y^2}{x^8} e^{x^2} \, dx dy$$

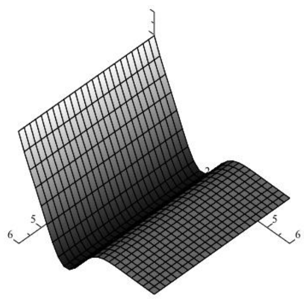
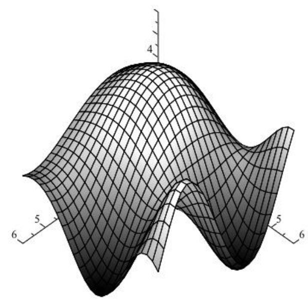
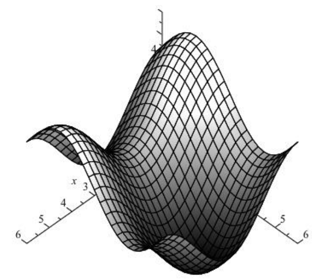
(a) $\boxed{\text{2 points}}$ Sketch the domain of integration on the plot below

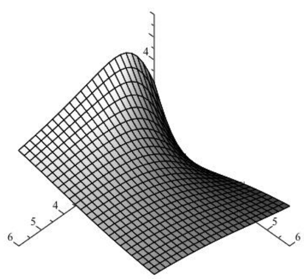
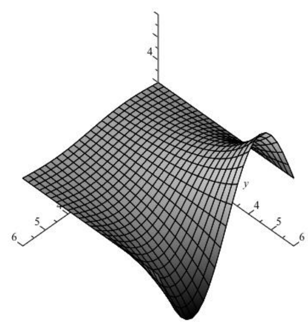
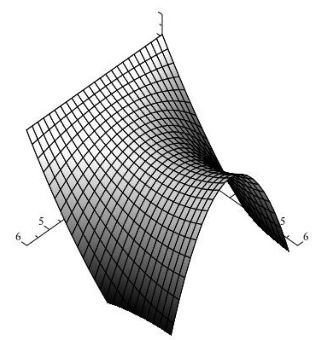


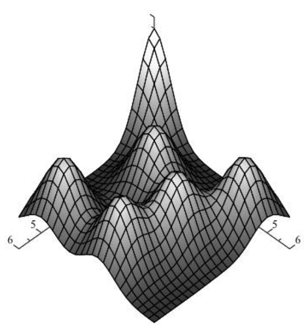
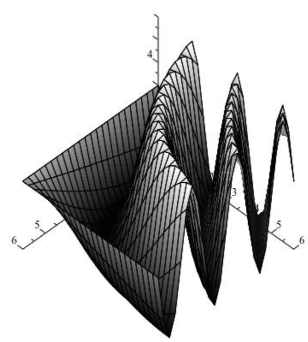
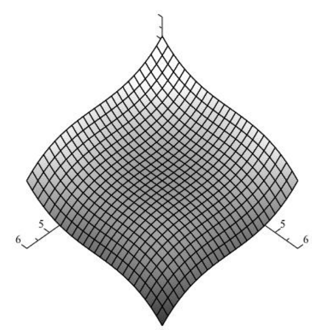
(b) $\boxed{\text{4 points}}$ Compute the integral.

4. $\boxed{\text{8 points}}$ Find the surface area of the part of the paraboloid $z = a^2 - x^2 - y^2$ which lies above the xy-plane.

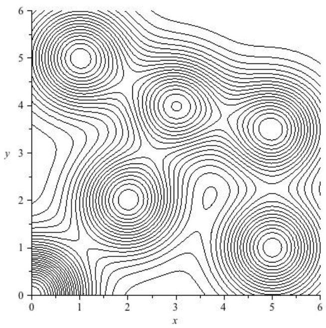
5. $\boxed{\text{9 points}}$ The axes of the nine graphs below are all oriented in the standard way: the positive x-axis is on the left, the positive y-axis is on the right, and the positive z-axis is upward. Put the letter of the corresponding contour plot from the next page in the box below each graph.

In the contour plots below, the *values* of the contours are evenly spaced. Nine of these twelve plots correspond to graphs on the previous page.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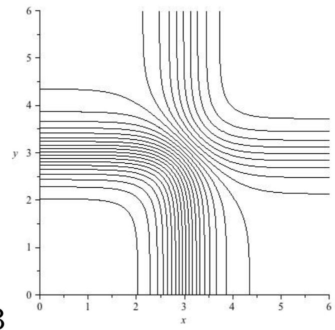
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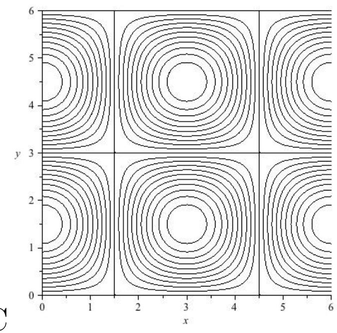
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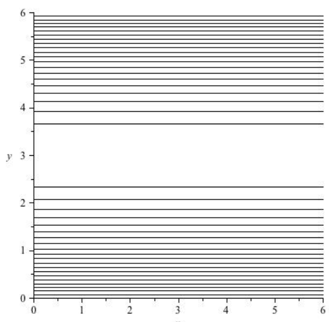
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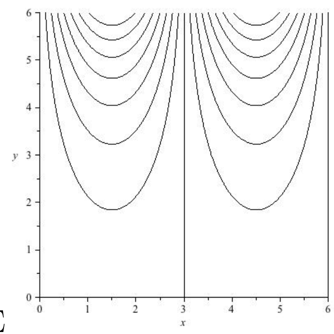
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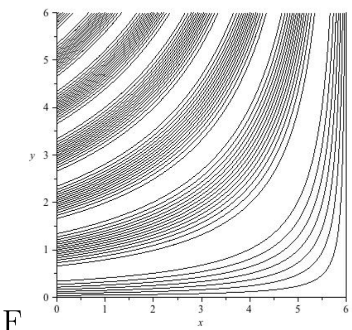
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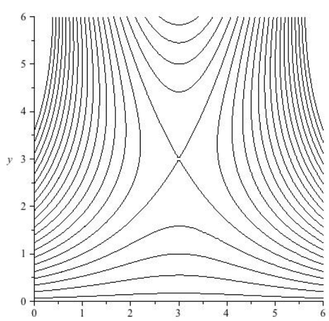
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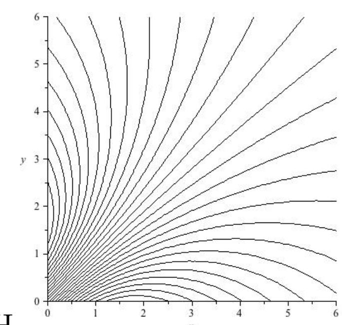
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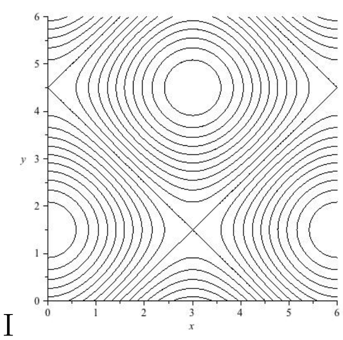
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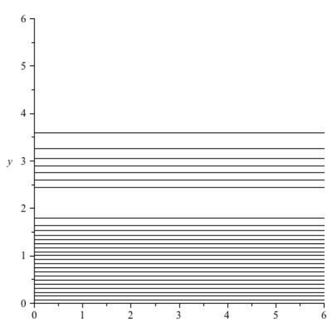
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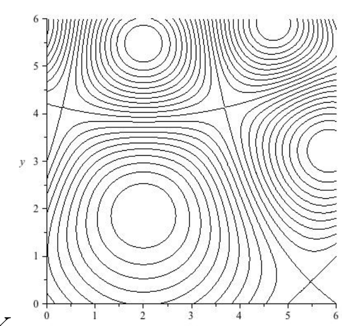
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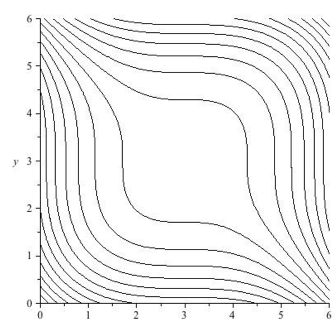
L

6. $\boxed{\text{12 points}}$ Let E be the tetrahedron with vertices $(0,-1,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. Compute the integral

$$\iiint_E z\,dV$$

7. $\boxed{\text{12 points}}$ Find the points on the ellipse $8x^2 + 12xy + 17y^2 = 100$ which are closest and farthest from the origin.

8. Consider the solid E which lies below the spherical surface $x^2 + y^2 + (z-1)^2 = 1$, and above the conical surface $z = \sqrt{x^2 + y^2}$.

   (a) $\boxed{\text{4 points}}$ Set up the integral $\iiint_E z\,dV$ in cylindrical coordinates. Do not evaluate (yet!).

   (b) $\boxed{\text{4 points}}$ Set up the integral $\iiint_E z\,dV$ in spherical coordinates. Do not evaluate (yet!).

   (c) $\boxed{\text{4 points}}$ Set up the integral $\iiint_E z\,dV$ in Cartesian coordinates. Do not evaluate (yet!).

(d) $\boxed{\text{4 points}}$ Evaluate the integral $\iiint_E z\, dV$.

(e) $\boxed{\text{4 points}}$ Find the coordinates of the center of mass of the solid E, assuming it has constant mass density.