The University of British Columbia
Final Examination - December 2007

Mathematics 253

Multivariable Calculus

Closed book examination Time: 2.5 hours

Name _____ **Signature** _____

Student Number _____

Circle your section no.: **101** Dr. Tsai **102** Dr. Graham **103** Dr. Froese

Special Instructions:
- Be sure that this examination has 10 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- Show all your work. Unsupported solutions deserve no mark.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate should be prepared to produce her/his library/AMS card upon request.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates.
- Smoking is not permitted during examinations.

1		20
2		12
3		10
4		12
5		12
6		12
7		10
8		12
Total		100

1. [20pt] For the following questions, fill in the answers in the boxes. No work need to be shown and no partial credit will be given.

(a) Find the angle between the two vectors $\mathbf{v}_1 = (1, 0, 1)$ and $\mathbf{v}_2 = (0, 1, 1)$.

Answer = $\boxed{}$

(b) Find the directional derivative of the function $f(x, y) = e^{x+y}$ along the direction of the vector $\mathbf{v} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ at $x = 0$, $y = 1$.

Answer = $\boxed{}$

(c) Find the tangent plane to the surface $x(\cos y)e^z = 1$ at the point $(1, 0, 0)$.

Answer = $\boxed{}$

1. (continue)

(d) What is $\dfrac{\partial^2}{\partial x^2}(\sin(x^2y^2z))$?

Answer = $\boxed{}$

(e) If $x(u,v) = u^2 - v^2$ and $y(u,v) = 2uv$. What is the Jacobian (stretching) factor if we change variables from (x,y) to (u,v)?

Answer = $\boxed{}$

2. [12pt] Given three points $A = (1, 2, 3)$, $B = (1, 3, 4)$, $C = (0, 1, 3)$.
(a) Find the equation of the plane E containing A, B, C.
(b) Find the area of the triangle with vertices A, B, C.

3. [10pt] If z is defined implicitly as a function of x and y by

$$xy^3 + x^2z^4 = 2,$$

what is $\dfrac{\partial z}{\partial x}$ when $x = y = z = 1$?

4. [12pt] Consider the function $f(x, y) = x - y + 2x^2 + 2y^2$.
(a) Find its critical points and values in the open disk $x^2 + y^2 < 1$.
(b) Find its maximum and minimum on the circle $x^2 + y^2 = 1$.
(c) Find its absolute maximum and minimum in the region $x^2 + y^2 \leq 1$.
You need to specify which value is maximum and which value is minimum.

5. [12pt] Find the closest points to the origin on the ellipse $x^2 + 4y^2 + 2x = 3$.

6. [12pt] Consider the tetrahedron bounded by the four planes

$$x = 0, \quad y = 0, \quad z = 0, \quad x + 2y + 3z = 6.$$

Find the limits of the integrals for its volume in the three different orders

$$\iiint dx\,dy\,dz, \quad \iiint dy\,dx\,dz, \quad \iiint dz\,dx\,dy.$$

7. [10pt] Compute $\iint_R x\,dA$ where the region R looks like a slice of pizza:

$$R: \quad x^2 + y^2 \leq 4, \quad 0 \leq y \leq x.$$

8. [12pt] Find the mass of the solid in between the two spheres centered at the origin with radii 1 and 2, above the xy-plane, and with density function z.
Hint: $\sin(2t) = 2\sin t \cos t$.