Final Exam
Math 230 Section 101 December 10th 2011

Name _____ **Student Number** _____

Signature _____

The exam is 150 minutes long and worth a total of 100 points. No books, notes or calculators may be used. **Show all of your work, simplify and justify your answers carefully**. You will be graded on the clarity of your exposition as well as the correctness of your answers.

Good luck.

UBC Rules governing examinations:

(a) Each candidate should be prepared to produce his/her UBCcard upon request for identification.

(b) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.

(c) No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour, *or the last 15 minutes* of the examination.

(d) Candidates guilty of any of the following or similar dishonest practices shall be immediately dismissed from the examination, and shall be liable to disciplinary action:

a) Making use of any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including phones), or other memory aid devices other than those authorized by the examiners.

b) Speaking or communicating with other candidates.

c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness will not be received.

(e) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	Points	Problem	Points
1		6	
2		7	
3		8	
4		9	
5		10	
Total			

1 (10 points)**.** *Definitions and theorems:*

(a) Define what is meant by *zero divisors* (mod m).

(b) Write down all the integers between 1 and 10 that are zero divisors (mod 24).

(c) State *Euler's characteristic formula theorem.*

(d) Calculate the number of vertices of a convex polyhedron with 3846 edges and 1264 faces.

2 (10 points)**.** *Twin primes question:* State the twin primes question, and give *five* pairs of numbers greater than 15 that support it (showing how).

3 (10 points)**.** *Transitivity theorem for congruence:* The **transitivity theorem for congruence** states that

$$\text{if } a \equiv b(\mod m) \text{ and } b \equiv c(\mod m) \text{ then } a \equiv c(\mod m).$$

Explain why this is true using the definition of congruence.

4 (10 points)**.** *Solving congruences:* Solve

$$50x \equiv 6(\mod 238).$$

What is the the *smallest positive* integer that is a solution to this?

5 (15 points)**.** *ASCII errors:* Explain why the ASCII will not detect TWO wrong digits.

6 (5 points)**.** *Calculating polyhedrons:* If a polyhedron has 8 vertices of degree 3, 7 vertices of degree 4, 6 vertices of degree 5, and 2 vertices of degree 12 then

how many faces does it have?

7 (10 points)**.** *Degree and edge theorem:* The **degree and edge theorem** states that for any convex polyhedron with e edges

$$\text{sum of the degrees } = 2e.$$

Explain why this is true.

8 (10 points)**.** *Error detecting codes:* There are 2 parts to this question.

(a) (5 points) *ISBN.* A friend spilled coffee over the back of your book obscuring an ISBN digit. Find the missing digit in its ISBN.

$$0 - 817 - 2575 * -X$$

(b) (5 points) *SIN.* Your friend's dog has been chewing their SIN card, and has chewed away the check digit. Compute the check digit for your friend's SIN.

$$483\ 199\ 87 * .$$

9 (10 points)**.** *Divisibility tests:* There are 3 parts to this question.

(a) State the divisibility test for 9.

(b) **Yes or no**: Is 21789834658 divisible by 9? Show your working.

(c) **Missing digit puzzle**: Your friend spilled coffee over your favourite number divisible by 9

$$3897 * 428152.$$

Find the missing digit and show your working.

10 (10 points)**.** *Block ciphers:* Consider the following table.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	0

Decrypt AOEMRVDIBRPVLLMU using the keyword SNOW. Show all your working.