**Be sure that this examination has 12 pages including this cover**

**The University of British Columbia**

Sessional Examinations - April 2013

**Mathematics 227**

*Advanced Calculus II*

Closed book examination

Time: $2\frac{1}{2}$ hours

**Name** ____________________ **Signature** ____________________

**Student Number**____________________ **Instructor's Name** ____________________

**Section Number** ____________________

## Special Instructions:

No books, notes, or calculators are allowed.

### Rules Governing Formal Examinations

**1.** Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

**2.** Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

**3.** No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

**4.** Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

**5.** Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

i. speaking or communicating with other examination candidates, unless otherwise authorized;

ii. purposely exposing written papers to the view of other examination candidates or imaging devices;

iii. purposely viewing the written papers of other examination candidates;

iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

**6.** Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

**7.** Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

| | | |
|---|---|---|
| 1 | | 14 |
| 2 | | 10 |
| 3 | | 12 |
| 4 | | 12 |
| 5 | | 12 |
| 6 | | 12 |
| 7 | | 12 |
| 8 | | 16 |
| Total | | 100 |

Marks

[14] **1.** A skier descends the hill $z = \sqrt{4 - x^2 - y^2}$ along a trail with parameterization

$$x = \sin(2\theta), \qquad y = 1 - \cos(2\theta), \qquad z = 2\cos\theta, \qquad 0 \le \theta \le \tfrac{\pi}{2}$$

Let $P$ denote the point on the trail where $x = 1$.

(a) Find the Frenet frame $\hat{\mathbf{T}}, \hat{\mathbf{N}}, \hat{\mathbf{B}}$ and the curvature $\kappa$ of the ski trail at the point $P$.

(b) The skier's acceleration at $P$ is $\mathbf{a} = (-2, 3, -2\sqrt{2})$. Find, at $P$,
(i) the rate of change of the skier's speed and
(ii) the skier's velocity (a vector).

Continued on page 3

Continued on page 4

[10] **2.** Let $\phi(x,y) = xy$ and let $\mathbf{F} = \boldsymbol{\nabla}\phi$.

(a) Find an equation for the field line of $\mathbf{F}$ which passes through the point $(3,2)$. Sketch it and verify that it also passes through $(3,-2)$.

(b) Find $\int_{(3,-2)}^{(3,2)} \mathbf{F} \cdot d\mathbf{r}$, where the line integral is along the field line of (a).

Continued on page 5

[12] **3.** Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force $\mathbf{F} = 2xy\,\hat{\boldsymbol{\imath}} + zy\,\hat{\boldsymbol{\jmath}} + x^2\,\hat{\mathbf{k}}$ on a particle as it moves from $(2, 0, 1)$ to $(0, 2, 5)$ along the curve $C$ of intersection (in the first octant $x, y, z \geq 0$) of the paraboloid $z = (x-1)^2 + y^2$ and the plane $z = 5 - 2x$.

Continued on page 6

[12] **4.** Find the surface area of the torus obtained by rotating the circle $(x-3)^2+z^2=4$, $y=0$ about the $z$–axis.

Continued on page 7

[12] **5.** Let $S$ be the portion of the sphere $x^2 + y^2 + z^2 = 36$ with $y + z \le 6$. Orient $S$ with normal pointing away from $(0, 0, 0)$. Let

$$\mathbf{F} = (y^2 z - x)\hat{\boldsymbol{\imath}} + (x + y + z)\hat{\boldsymbol{\jmath}} - x\hat{\mathbf{k}}$$

Evaluate $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$.

Continued on page 8

[12] **6.** Let $\mathbf{F} = (e^{x^2}+y)\,\hat{\boldsymbol{\imath}}+(\sin y^3+xz)\,\hat{\boldsymbol{\jmath}}+z^2\,\hat{\mathbf{k}}$. Evaluate $\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$ where $\mathcal{C}$ is the curve $\left\{\begin{matrix} x^2+y^2=10 \\ x+y+z=4 \end{matrix}\right\}$ with positive (i.e. counter-clockwise) orientation as viewed from high on the $z$-axis.

Continued on page 9

[12] **7.** A pile of wet sand having total volume $5\pi$ covers the disk $z = 0$, $x^2 + y^2 \le 1$. Call the top surface of the pile of sand $S$. The momentum of water vapour is given by the vector field

$$\mathbf{F} = \boldsymbol{\nabla}\phi + \mu\boldsymbol{\nabla} \times \mathbf{G}$$

where $\phi$ is the water concentration, $\mathbf{G}$ is the temperature gradient and $\mu$ is a constant. Suppose that $\phi = x^2 - y^2 + z^2$ and $\mathbf{G} = \frac{1}{3}\big( - y^3\hat{\boldsymbol{\imath}} + x^3\hat{\boldsymbol{\jmath}} + z^3\hat{\mathbf{k}}\big)$. Find the flux of $\mathbf{F}$ upward through $S$.

Continued on page 10

Continued on page 11

[16] **8.** Say whether each of the following statements is true or false and *explain why.* You may assume that the curves, surfaces and functions are all sufficiently smooth.

(a) Let $\phi$ be a real–valued function. Suppose that

$$\iint_S \nabla\phi \cdot \hat{\mathbf{n}}\, dS = 0$$

for every closed oriented surface $S$. Here $\hat{\mathbf{n}}$ is the outward unit normal to $S$. Then $\phi$ is a constant.

(b) If $\mathbf{F}$ and $\mathbf{G}$ are two vector fields defined in $\mathbb{R}^3$ and $\int_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS = \int_S \mathbf{G} \cdot \hat{\mathbf{n}}\, dS$ for every orientable surface $S$, then $\mathbf{F} = \mathbf{G}$.

(c) Let $\mathbf{F}$ be a vector field in the $xy$-plane which is perpendicular to $\mathbf{r} = (x, y)$ at all points. Then each flow line of $\mathbf{F}$ is contained in a circle.

(d) Let $\mathbf{F}$ be a vector field in the $xy$-plane which is perpendicular to $\mathbf{r} = (x, y)$ at all points. Then the unit circle is a flow line of $\mathbf{F}$.

Continued on page 12

**The End**