Marks

[10] **1.** A curve in $\mathbf{R}^3$ is given by the parametric equation $\mathbf{x}(t) = (e^t, e^{-t}, \sqrt{2}t)$.

(a) (4 marks) Find the length of the curve between $t = 0$ and $t = 1$.

(b) (6 marks) Find the curvature at a general point $\mathbf{x}(t)$.

Continued on page 3

[12] **2.** Let $f(x, y) = xe^y - y^2e^x$.

(a) (6 marks) Find the first- and second-order Taylor polynomials $P_1(x, y)$ and $P_2(x, y)$ at $(0, 1)$. (It is not necessary to simplify your answers.)

(b) (6 marks) Prove that $f(x, y) - P_1(x, y)$, where P_1 is the first-order Taylor polynomial from (a), has a local maximum at $(0, 1)$. (Hints: (1) $e \approx 2.72$, (2) you have done most of the necessary calculations in (a).)

Continued on page 4

[8] **3.** Find the minimum distance from the origin to the surface $z(3x + 4y) = 20$.

Continued on page 5

[10] **4.** Evaluate the following integrals:

(a) (5 marks) $\int\int_D \cos(x^2)dA$, where D is the triangle in the xy-plane with vertices $(0,0)$, $(2,0)$, $(2,2)$;

(b) (5 marks) $\int\int\int_W xz\,dV$, where W is the bounded solid enclosed by the planes $z=0$, $z=2$, $y=0$, $y=x$, and the cylinder $x^2+y^2=1$.

Continued on page 6

[12] **5.** Evaluate the line integrals below. (Use any methods you like.)

(a) (6 marks) $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + 4z\mathbf{k}$ and $\mathbf{x}$ is the parametrized curve $(\cos t + \sin t, \cos t - \sin t, t)$, $0 \leq t \leq 1$.

(b) (6 marks) The (outward) flux of $\mathbf{F}(x, y) = (x^3 + \sin y)\mathbf{i} + e^{x+y}\mathbf{j}$ across the boundary of the rectangle $0 \leq x \leq 1, \ 0 \leq y \leq 2$ in the xy-plane.

Continued on page 7

[12] **6.**

(a) (4 marks) Find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$, where $\mathbf{F}(x, y) = (x^2 + y^2)\mathbf{i} + 2xy\mathbf{j}$.

(b) (4 marks) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is any oriented piecewise C^1 curve from $(1, 2)$ to $(3, 4)$ and $\mathbf{F}$ is the vector field in (a).

(c) (4 marks) Let $\mathbf{F} = \nabla f$ be a conservative vector field (not necessarily the same as in (a)-(b)), and let $\mathbf{x}(t)$ be a flow line of $\mathbf{F}$. Prove that $\frac{d}{dt} f(\mathbf{x}(t)) \geq 0$.

Continued on page 8

[8] **7.** Let $\mathbf{F} = (x + z)\mathbf{i} + (y + 2z)\mathbf{j} + (2x + 3y)\mathbf{k}$. What are the possible values of $\int_C \mathbf{F} \cdot d\mathbf{s}$, if C is a circle of radius r contained in a plane $x + 3y - z = a$?

Continued on page 9

[16] **8.** Let $\mathbf{X}$ be the parametrized surface $\mathbf{X}(s,t) = (t\cos s, t\sin s, 2t)$, $0 \le s \le \pi/2$, $1 \le t \le 2$. Evaluate the following integrals:

(a) (8 marks) $\int\int_{\mathbf{X}} z^2 dS$,

(b) (8 marks) $\int\int_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F} = y^2\mathbf{i}$.

Continued on page 10

[12] **9.** Let $\omega = (x+z)dx \wedge dy + (y-x)dy \wedge dz$.

(a) (4 marks) Compute $d\omega$. Simplify your answer.

(b) (8 marks) Find $\int_{\mathbf{X}} \omega$, if $\mathbf{X}(s,t) = (t+s, t, s^2)$, $-1 \leq s \leq 1$, $0 \leq t \leq 1$.

The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - April 2009

Mathematics 227

Advanced Calculus II

Closed book examination — Time: 2.5 hours

Print Name ______________ **Signature** ______________

Student Number ______________ **Instructor's Name** ______________

Section Number ______________

Special Instructions:

No calculators, notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

Rules governing examinations

1		10
2		12
3		8
4		10
5		12
6		12
7		8
8		16
9		12
Total		100