**The University of British Columbia**

Final Examination - December 9, 2006

**Mathematics 226**

Section 101

Instructor: C. Lamb

Closed book examination | Time: 2.5 hours

**Last Name** ____________ **First** ________ **Signature** ____________

**Student Number** ____________

**Special Instructions:**

No notes, books or calculators are to be used. No credit will be given for the correct answer without the (correct) accompanying work. Use the back of the pages if you need extra space.

**Rules governing examinations**

• Each candidate must be prepared to produce, upon request, a UBCcard for identification.

• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

• Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

• Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

• Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

| | | |
|---|---|---|
| 1 | | 15 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 15 |
| 7 | | 15 |
| 8 | | 15 |
| Total | | 100 |

[15] **1.** Let $z = f(x, y)$ be a differentiable function on $\mathbf{R}^2$ such that $f(1, 2) = 3$, $f(1.2, 2.3) = 3.4$ and $f(0.9, 2.1) = 3.2$.

(a) Estimate $\dfrac{\partial z}{\partial x}(1, 2)$ and $\dfrac{\partial z}{\partial y}(1, 2)$. [10pt]

(b) Estimate the value of the directional derivative of $z = f(x, y)$ at the point $(1, 2)$ as you move towards the point $(2, 3)$. [5pt]

Extra blank page

[10] **2**. Let $z = f(x, y)$ be a differentiable function on $\mathbf{R}^2$, $x = (s^2 + t^2)/2$ and $y = (s^2 - t^2)/2$. Express the quantity

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

in terms of $s$, $t$, $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

[10] **3**. Consider the surface defined by the equation

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = 2,$$

where $x > 0$, $y > 0$ and $z > 0$. Show that the sum of the $x$-, $y$- and $z$-intercepts of any tangent plane to this surface is a constant, and determine the value of this constant. Hint: Start out by showing that if $(x_0, y_0, z_0)$ is any point on the surface, and hence $\sqrt{x_0} + \sqrt{y_0} + \sqrt{z_0} = 2$, then the vector $\left(\frac{1}{2\sqrt{x_0}}\right)\mathbf{i} + \left(\frac{1}{2\sqrt{y_0}}\right)\mathbf{j} + \left(\frac{1}{2\sqrt{z_0}}\right)\mathbf{k}$ is a normal vector to that surface at $(x_0, y_0, z_0)$.

[10] **4**. Let the temperature at a point $(x, y, z)$ be given by $w = x^3y^2z$. Find the point on the plane $2x+2y+z = 24$ where the temperature is a maximum. You do not need to justify that your answer actually gives the maximum. Hint: You may assume in your calculations that $x \neq 0$, $y \neq 0$ and $z \neq 0$ since if $x$, $y$ or $z$ equals 0, then $w = 0$ and this will not be the maximum temperature.

[10] **5**. Evaluate the interated double integral

$$\int_{y=0}^{y=\sqrt{\pi}} \int_{x=y}^{x=\sqrt{\pi}} \sin(x^2)\, dx\, dy.$$

[15] **6**. Let $W$ be the 3-dimensional solid defined by the inequalities $x \geq 0$, $y \geq 0$, $z \geq 0$, $x + y^2 \leq 1$ and $x - y + z \leq 1$.

(a) Draw a sketch of $W$. Be sure to show the units on the axes. [5pt]

(b) Find the volume of $W$. [10pt]

[15] **7**. Let $W$ be the 3-dimensional solid defined by the inequalities $x^2 + y^2 \leq 1$ and $0 \leq z \leq 2 - x^2 - y^2$.

(a) Draw a sketch of $W$. Be sure to show the units on the axes. [5pt]

(b) Use cylindrical coordinates to evaluate $\iiint_W z dV$. [10pt]

[15] **8**. Let $z = f(x, y)$ be defined by

$$z = \begin{cases} \dfrac{y^5}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

(a) Use the definition of partial derivatives as limits to calculate $\dfrac{\partial z}{\partial x}(0, 0)$ and $\dfrac{\partial z}{\partial y}(0, 0)$. [5pt]

(b) Is $z = f(x, y)$ differentiable at $(0, 0)$? If you think that this is the case, then give a formal $\epsilon - \delta$ proof to justify your belief. Otherwise, indicate clearly why you believe that $z = f(x, y)$ is not differentiable at $(0, 0)$. [10pt]

**The End**