Marks

[12] **1.** (6 marks for each part)

(a) Prove that the line given by the parametric equations $x = 1 + 4t$, $y = 2 - t$, $z = -3t$, is parallel to the plane $2x + 5y - z = 4$.

(b) Find the distance between the plane and the line in (a).

Continued on page 3

[10] **2.** Find all points on the surface $3x^2 - y^2 + 2z^2 = 1$ where the tangent plane is parallel to both of the vectors $(2, 2, 1)$ and $(4, 1, -5)$.

Continued on page 4

[10] **3.**

(a) (6 marks) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(x, y) = (1, 0)$, if $z = f(e^{x+2y}, \sin(xy), e^{x-y})$ and $f : \mathbf{R}^3 \to \mathbf{R}$ is a function of class C^1 such that $f(e, 0, e) = (1, 1, 2)$. $\nabla f(e, 0, e) = (3, -1, 2)$. (Use the Chain Rule).

(b) (4 marks) If $\mathbf{F}(x, y) = \begin{pmatrix} z \\ z^2 \end{pmatrix}$, where z is as in (a), find $D\mathbf{F}(1, 0)$.

Continued on page 5

[13] **4.**

(a) (10 marks) Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^4 + y^4 - 4xy + 6$.

(b) (3 marks) Does the function in (a) have a global maximum or minimum? Explain why or why not.

Continued on page 6

[10] **5.** The plane $x + 2y + z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse which are nearest to and farthest from the origin.

Continued on page 7

[18] **6.** In each part of this problem, provide a precise definition of the word or phrase in boldface. Let

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Prove that f is **continuous** at $(0,0)$. (Hint: use polar coordinates.)

(b) If $\mathbf{u}$ is a unit vector, find the **directional derivative** $D_{\mathbf{u}}f(0,0)$ directly from the definition.

(continued on next page)

(c) Is f **differentiable** at $(0,0)$? Explain why or why not.

[5] **7.** Let $f : \mathbf{R}^n \to \mathbf{R}$ be a function of class C^1 such that

$$f(t\mathbf{x}) = t^a f(\mathbf{x}) \text{ for all } \mathbf{x} \in \mathbf{R}^n, t > 0$$

for some fixed $a \in \mathbf{R}$ (such functions are called *homogeneous of degree a*). Prove that

$$\mathbf{x} \cdot \nabla f(\mathbf{x}) = af(\mathbf{x}).$$

(Hint: for fixed $\mathbf{x}$, differentiate $f(t\mathbf{x})$ with respect to t.)

Continued on page 9

[12] **8.** Evaluate the following integrals. (6 marks for each part)

(a) $\int\int_D x dA$, if D is the region bounded by the parabola $y^2 - x - 5 = 0$ and the line $x + 2y = 3$. (Hint: pay attention to the choice of the order of integration.)

(a) $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx$. (Hint: reverse the order of integration.)

Continued on page 10

[10] **9.** (5 marks for each part) Let R be the solid region in $\mathbf{R}^3$ bounded by the planes $x = 0$, $y = 0$, $y = 4 - x$, and the surface $z = 4 - x^2$. Write $\int\int\int_R f(x, y, z)dV$ as an iterated integrals where the order of integration is as indicated below (i.e. find the limits of integration):

(a) $\int\int\int f(x, y, z)dzdydx$

(b) $\int\int\int f(x, y, z)dydxdz$

The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - December 2005

Mathematics 226

Advanced Calculus I

Closed book examination | Time: 2.5 hours

Print Name ______________________ **Signature** ___________________________

Student Number ___________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

No calculators, notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

1		12
2		10
3		10
4		13
5		10
6		18
7		5
8		12
9		10
Total		100

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1. Each candidate should be prepared to produce his library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

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(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.