Be sure that this examination has 2 pages.

The University of British Columbia

Sessional Examinations - December 2001

Mathematics 226

Advanced Calculus I

Closed book examination | Time: $2\frac{1}{2}$ hours

Special Instructions: No notes or calculators are allowed.

Marks

[10] **1.** Define carefully:

(a) Differentiability at $(0,0)$ of a function $f : \mathbf{R}^2 \to \mathbf{R}$.

(b) The integral, $\int_R f dV$, of a function $f : R \to \mathbf{R}$, where $R = [0,1] \times [0,1] \times [0,1]$.

[15] **2.** Give examples of the following. Briefly justify your examples.

(a) A continuous function $f : D \to \mathbf{R}$ which has no absolute maximum. Here $D = \{(x,y) : x^2 + y^2 < 1\}$ is the open unit disk.

(b) A subset of $\mathbf{R}^2$ which is neither open or closed.

(c) A discontinuous bounded function $f : [0,1]^2 \to \mathbf{R}$, which is integrable over $[0,1]^2$.

[8] **3.** Find the equation of the plane which contains $(1,2,3)$ and $(4,6,7)$ and is perpendicular to the plane $3x + 2y + z = 1$.

[10] **4.** Find and classify all critical points of $f(x,y) = x^4 + y^4 - 4xy^2$.

[12] **5.** Assume temperature (in degrees Celsius) is a C^1 function $T : \mathbf{R}^2 \to \mathbf{R}$ and $T(0,0) = 10$. A particle at $(0,0)$ travelling with speed 1 unit/second in the direction of $\mathbf{i}$ notes an increase of temperature at a rate of $.3^o C$ per second and the same particle notes an increase of temperature of $.1^o C$ per second when it heads in the direction $.6\mathbf{i} + .8\mathbf{j}$.

(a) In what direction should the particle head if it wants to try to maintain its current temperature.

(b) Find $\lim_{h \to 0} \frac{T(2h,h) - 10}{h}$.

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[15] **6.** (a) Briefly explain why the function $f(x,y,z) = x + y^2 z$ on its domain $D = \{(x,y,z) : 2x^2 + y^2 + z^2 \le 1\}$ has an absolute maximum and minimum.

(b) Find all absolute minima and maxima of the function in (a).

[16] **7.** Evaluate:

(a) The mass of a triangular plate with vertices at $(0,0)$, $(1,1)$ and $(1,3)$, and density $f(x,y) = xy$.

(b) $\int_0^1 \left(\int_0^{1-x} \left(\int_y^1 \frac{e^{z^2}}{2-z} dz \right) dy \right) dx$.

[10] **8.** Let $f(x,y) = (2x + xy, x^4 + e^y - 1)$ and $g = f \circ f \circ f \circ f$. Find $Dg(0,0)$.

[12] **9.** (a) Define $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \in [0,1]^2 \text{ and } (x,y) \neq 0 \\ 0 & \text{otherwise.} \end{cases}$ Show that for each fixed $y \in [0,1]$, $f(x,y)$ is a continuous function of $x \in [0,1]$ and for each fixed $x \in [0,1]$ $f(x,y)$ is a continuous function of $y \in [0,1]$, but f is not a continuous function on $[0,1]^2$.

(b) We say $f : [0,1]^2 \to \mathbf{R}$ is continuous in x uniformly in y iff for each $x_0 \in [0,1]$, for all $\varepsilon > 0$ there is a $\delta > 0$ such that if $x \in [0.1]$ and $|x - x_0| < \delta$, then for all $y \in [0,1]$, $|f(x,y) - f(x_0,y)| < \varepsilon$. Suppose $f : [0,1]^2 \to \mathbf{R}$ is continuous in x uniformly in y and for each $y_0 \in [0,1]$ is $x \mapsto f(x,y)$ is continuous on $[0,1]$. Prove that f is continuous on $[0,1]^2$.

[108] **Total Marks**

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