FINAL EXAM

Math 220 Section 101

December 7, 2005

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Signature: ______________________________

The exam is worth a total of 100 points with duration 2.5 hours. No books, notes or calculators are allowed. Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/AMS card upon request.
(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.
(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Problem 1 (15 points). Determine whether the following series converge or diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(3n)^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\left(1 + \frac{1}{2n}\right)^n - 1 \right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + (-1)^n}{\sqrt{n}}$

Problem 2 (10 points). Prove the following limits exist and then evaluate them.

(a) $\lim\limits_{n\to\infty}\left(\sqrt{4n^4+n}-2n^2\right)$

(b) $\lim\limits_{n\to\infty} x_n$, where $x_1=3, x_{n+1}=\sqrt{2x_n+15}$

Problem 3 (18 points). Mark each statement True or False. If True, give a proof. If False, give a counter-example.

(a) Let S, T be two nonempty subsets of $(0, 1]$ and $\inf T \in (0, 1]$. Then $\sup \{\frac{s}{t} | s \in S, t \in T\}$ exists as a real number.

(b) If $f : \mathbb{R} \to \mathbb{Q}$ is a function, then f is not injective.

(c) Let S be a subset of $\mathbb{R}$ and let a by an upper bound of S. If $a \in S$, then S is not open.

Problem 4 (12 points) Let T be a collection of nonempty sets. Define a relation $\mathbf{R}$ on T by $A\mathbf{R}B$ where $A \in T, B \in T$ if and only if there exists a bijective function from A to B. Prove that $\mathbf{R}$ is an equivalent relation on T.

Problem 5 (10 points) Let $g : \mathbb{R} \to \mathbb{R}$ be a function which satisfies $|g(x) - g(y)| \leq 2|x - y|$ for all real numbers x and y. Suppose that $f : \mathbb{N} \to \mathbb{R}$ is a function and $(f(n))$ is a Cauchy sequence. Show that the sequence $(g \circ f(n))$ converges.

Problem 6 (12 points) If (a_n) is an increasing sequence with $\lim_{n\to\infty} a_n = 0$, prove that $\sum_{n=1}^{\infty} a_n^n$ converges.

Problem 7 (12 points) Let (a_n) be a bounded sequence in $\mathbb{R}$. Show that

$$\limsup a_n = -\liminf(-a_n).$$

Problem 8 (11 points) Let S_1 and S_2 be two nonempty subsets of $\mathbb{R}$. Let A_1, A_2, A_3 be the sets of accumulation points of $S_1, S_2, S_1 \cup S_2$ respectively. Prove that $A_3 = A_1 \cup A_2$.