

The University of British Columbia
 Final Examination - December 6, 2014
Mathematics 200

Closed book examination

Time: 2 hours 30 minutes

Last Name: _____ **First:** _____

Student Number: _____

Section (check one): 101 (MWF 9-10, Peterson) 102 (MWF 11-12, Fraser)
 103 (MWF 11-12, Nguyen) 104 (MWF 1-2, Liu)
 105 (TuTh 9:30-11, Roe) 107 (TuTh 3:30-5, Roe)

Special Instructions:

- Be sure that this examination has 13 pages. Write your name on top of each page.
- No books, notes, calculators, or any other aids are allowed.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		7
2		10
3		10
4		8
5		14
6		14
7		14
8		14
9		9
Total		100

1. Suppose that $f(x, y, z)$ is a function of three variables and let $\mathbf{u} = \frac{1}{\sqrt{6}}\langle 1, 1, 2 \rangle$ and $\mathbf{v} = \frac{1}{\sqrt{3}}\langle 1, -1, -1 \rangle$ and $\mathbf{w} = \frac{1}{\sqrt{3}}\langle 1, 1, 1 \rangle$. Suppose that at a point (a, b, c) ,

$$D_{\mathbf{u}}f = 0$$

$$D_{\mathbf{v}}f = 0$$

$$D_{\mathbf{w}}f = 4.$$

Find ∇f at (a, b, c) .

2. Let $f(u, v)$ be a differentiable function of two variables, and let z be a differentiable function of x and y defined implicitly by $f(xz, yz) = 0$. Show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = -z.$$

3. Let $z = f(x, y)$ be given implicitly by

$$e^z + yz = x + y.$$

(a) Find the differential dz .

(b) Use linear approximation at the point $(1, 0)$ to approximate $f(0.99, 0.01)$.

4. Let S be the surface $z = x^2 + 2y^2 + 2y - 1$. Find all points $P(x_0, y_0, z_0)$ on S with $x_0 \neq 0$ such that the normal line at P contains the origin $(0, 0, 0)$.

5. Let $f(x, y) = xy(x + y - 3)$.

- (a) Find all critical points of f , and classify each one as a local maximum, a local minimum, or saddle point.

- (b) Find the location and value of the absolute maximum and minimum of f on the triangular region $x \geq 0, y \geq 0, x + y \leq 8$.

6. In the xy -plane, the disk $x^2 + y^2 \leq 2x$ is cut into 2 pieces by the line $y = x$. Let D be the larger piece.

(a) Sketch D including an accurate description of the center and radius of the given disk. Then describe D in polar coordinates (r, θ) .

(b) Find the volume of the solid below $z = \sqrt{x^2 + y^2}$ and above D .

7. The density of hydrogen gas in a region of space is given by the formula

$$\rho(x, y, z) = \frac{z + 2x^2}{1 + x^2 + y^2}.$$

(a) At $(1, 0, -1)$, in which direction is the density of hydrogen increasing most rapidly?

- (b) You are in a spacecraft at the origin. Suppose the spacecraft flies in the direction of $\langle 0, 0, 1 \rangle$. It has a disc of radius 1, centred on the spacecraft and deployed perpendicular to the direction of travel, to catch hydrogen. How much hydrogen has been collected by the time that the spacecraft has traveled a distance 2? [You may use the fact that $\int_0^{2\pi} \cos^2 \theta \, d\theta = \pi$.]

8. Consider the iterated integral

$$I = \int_{-a}^0 \int_{-\sqrt{a^2-x^2}}^0 \int_0^{\sqrt{a^2-x^2-y^2}} (x^2 + y^2 + z^2)^{2014} dz dy dx$$

where a is a positive constant.

(a) Write I as an iterated integral in cylindrical coordinates.

(b) Write I as an iterated integral in spherical coordinates.

(c) Evaluate I using whatever method you prefer.

9. Let E be the region bounded by $z = 2x$, $z = y^2$, and $x = 3$. The triple integral $\iiint_E f(x, y, z) dV$ can be expressed as an iterated integral in the following three orders of integration. Fill in the limits of integration in each case. No explanation required.

$$\int_{y=}^{y=} \int_{x=}^{x=} \int_{z=}^{z=} f(x, y, z) dz dx dy$$

$$\int_{y=}^{y=} \int_{z=}^{z=} \int_{x=}^{x=} f(x, y, z) dx dz dy$$

$$\int_{z=}^{z=} \int_{x=}^{x=} \int_{y=}^{y=} f(x, y, z) dy dx dz$$