The University of British Columbia

Final Examination - April 16, 2011

Mathematics 200

Closed book examination Time: 2.5 hours

Last Name: ____________________ , **First:** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

- No books, notes or calculators are allowed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		10
4		14
5		16
6		14
7		12
8		14
Total		100

[10] **1.** Let $A = (2, 3, 4)$ and let L be the line given by the equations $x + y = 1$ and $x + 2y + z = 3$.

(a) Write a vector equation for L.

(b) Write an equation for the plane containing A and perpendicular to L.

(c) Write an equation for the plane containing A and L.

[10] **2.** According to van der Waal's equation, a gas satisfies the equation

$$(pV^2 + 16)(V - 1) = TV^2,$$

where p, V and T denote pressure, volume and temperature respectively. Suppose the gas is now at pressure 1, volume 2 and temperature 5. Find the approximate change in its volume if p is increased by .2 and T is increased by .3.

[10] **3.** Suppose that $w = f(xz, yz)$, where f is a differentiable function. Show that

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = z\frac{\partial w}{\partial z}.$$

[14] **4.** Let

$$f(x, y, z) = (2x + y)e^{-(x^2+y^2+z^2)},$$
$$g(x, y, z) = xz + y^2 + yz + z^2.$$

(a) Find the gradients of f and g at (0,1,-1).

(b) A bird at (0,1,-1) flies at speed 6 in the direction in which $f(x, y, z)$ increases most rapidly. As it passes through (0,1,-1), how quickly does $g(x, y, z)$ appear (to the bird) to be changing?

(c) A bat at (0,1,-1) flies in the direction in which $f(x, y, z)$ and $g(x, y, z)$ do not change, but z increases. Find a vector in this direction.

[16] **5.** Let $h(x,y) = y(4 - x^2 - y^2)$.

(a) Find and classify the critical points of $h(x,y)$ as local maxima, local minima or saddle points.

(b) Find the maximum and minimum values of $h(x,y)$ on the disk $x^2 + y^2 \leq 1$.

(c) Find the maximum value of $f(x,y,z) = xyz$

on the ellipsoid $g(x,y,z) = x^2 + xy + y^2 + 3z^2 = 9$.

Specify all points at which this maximum value occurs.

[14] **6.** Consider

$$J = \int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{y}{x} e^{x^2+y^2}\, dx\, dy.$$

(a) Sketch the region of integration.

(b) Reverse the order of integration.

(c) Evaluate J by using polar coordinates.

[12] **7.** Let E be the portion of the first octant which is above the plane $z = x + y$ and below the plane $z = 2$. The density in E is $\rho(x, y, z) = z$. Find the mass of E.

[14] **8.** Let E be the "ice cream cone" $x^2 + y^2 + z^2 \leq 1$, $x^2 + y^2 \leq z^2$, $z \geq 0$. Consider

$$J = \iiint_E \sqrt{x^2 + y^2 + z^2}\,dV.$$

(a) Write J as an iterated integral, with limits, in cylindrical coordinates.

(b) Write J as an iterated integral, with limits, in spherical coordinates.

(c) Evaluate J.