The University of British Columbia
Final Examination - December, 2010
Mathematics 200

1. Let $z = f(x, y) = \frac{2y}{x^2+y^2}$.
(a) Sketch the level curves of $f(x, y)$
(b) Find the tangent plane and normal line to the surface $z = f(x, y)$ at $(x, y) = (-1, 2)$.
(c) Find an approximate value for $f(-0.8, 2.1)$.

2. Suppose the function $T = F(x, y, z) = 3 + xy - y^2 + z^2 - x$ describes the temperature at a point (x, y, z) in space, with $F(3, 2, 1) = 3$.
(a) Find the directional derivative of T at $(3, 2, 1)$, in the direction of the vector $\mathbf{j} + 2\mathbf{k}$ $= \langle 0, 1, 2\rangle$
(b) At the point $(3, 2, 1)$, in what direction does the temperature *decrease* most rapidly?
(c) Moving along the curve given by $x = 3e^t$, $y = 2\cos t$, $z = \sqrt{1+t}$, find $\frac{dT}{dt}$, the rate of change of temperature with respect to t, at $t = 0$.
(d) Suppose $\mathbf{i}+5\mathbf{j}+a\mathbf{k}$ is a vector that is tangent to the temperature level surface $T(x, y, z) = 3$ at $(3, 2, 1)$. What is a?

3. Find $\frac{\partial U}{\partial T}$ and $\frac{\partial T}{\partial V}$ at $(1, 1, 2, 4)$ if (T, U, V, W) are related by

$$(TU - V)^2 \ln(W - UV) = \ln 2.$$

4 (a) For the function

$$z = f(x, y) = x^3 + 3xy + 3y^2 - 6x - 3y - 6.$$

Classify [as local maxima, minima, or saddle points] all critical points of $f(x, y)$

(b) Find the point $P = (x, y, z)$ (with x, y and $z > 0$) on the surface $x^3y^2z = 6\sqrt{3}$ that is closest to the origin.

5. (a) D is the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.
Sketch D and evaluate J where

$$J = \iint_D 3y\, dA$$

(b) Sketch the region of integration and then evaluate the integral I :

$$I = \int_0^4 \int_{\frac{1}{2}\sqrt{x}}^1 e^{y^3}\, dy\, dx.$$

6. Let E be the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$. Find the centroid of E.

7. Let T denote the tetrahedron bounded by the coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $x + y + z = 1$.
Compute

$$K = \iiint_T \frac{1}{(1 + x + y + z)^4}\, dV.$$

8. Evaluate $W = \iiint_Q xz\; dV$, where Q is an eighth of the sphere $x^2 + y^2 + z^2 \leq 9$ with $x, y, z \geq 0$.