

The University of British Columbia

Final Examination - April, 2010

Mathematics 200

Closed book examination

Time: 2.5 hours

Last Name: \_\_\_\_\_, First: \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Section \_\_\_\_\_

Special Instructions:

No books, notes or calculators are allowed. Use backs of sheets if extra space needed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCCard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  - (b) Speaking or communicating with other candidates.
  - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		12
2		14
3		12
4		10
5		13
6		13
7		13
8		13
Total		100

[12] 1.(a) A surface  $z(x, y)$  is defined by  $zy - y + x = \ln(xyz)$ .

(i) Compute  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  in terms of  $x, y, z$ .

(ii) Evaluate  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at  $(x, y, z) = (-1, -2, 1/2)$ .

1.(b) A surface  $z = f(x, y)$  has derivatives  $\frac{\partial f}{\partial x} = 3$  and  $\frac{\partial f}{\partial y} = -2$  at  $(x, y, z) = (1, 3, 1)$ .

(i) If  $x$  increases from 1 to 1.2, and  $y$  decreases from 3 to 2.6,

find the change in  $z$  using a linear approximation.

(ii) Find the equation of the tangent plane to the surface at the point  $(1, 3, 1)$ .

[14] **2.** (i) For the function

$$z = f(x, y) = x^3 + 3xy + 3y^2 - 6x - 3y - 6.$$

Find and classify [as local maxima, local minima, or saddle points] all critical points of  $f(x, y)$

2 (ii) The images below depict level sets  $f(x, y) = c$  of the functions in the list at heights  $c = 0, 0.1, 0.2, \dots, 1.9, 2$ .

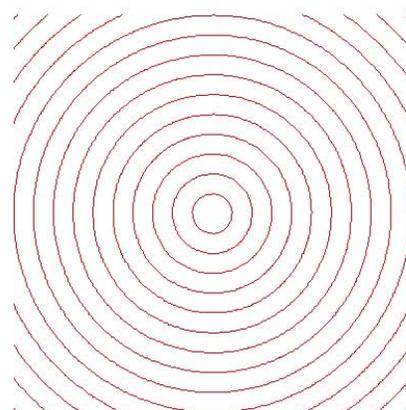
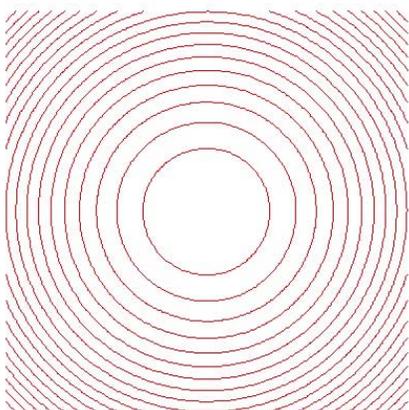
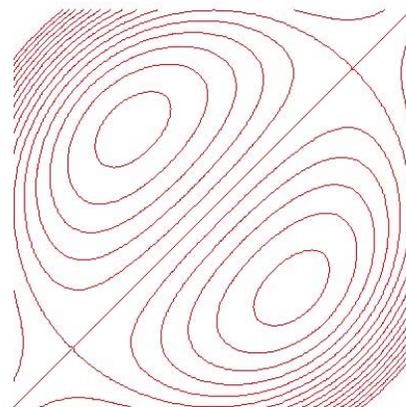
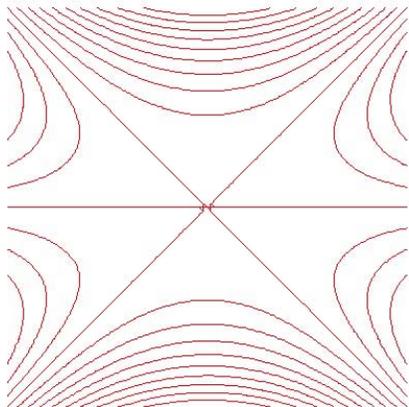
Label the pictures with the corresponding function and mark the critical points in each picture. (Note that in some cases, the critical points might not be drawn on the images already. In those cases you should add them to the picture.)

(a)  $f(x, y) = (x^2 + y^2 - 1)(x - y) + 1$

(b)  $f(x, y) = \sqrt{x^2 + y^2}$

(c)  $f(x, y) = y(x + y)(x - y) + 1$

(d)  $f(x, y) = x^2 + y^2$



[12] **3.** The temperature  $T(x, y)$  at a point of the  $xy$ -plane is given by

$$T(x, y) = ye^{x^2}.$$

A bug travels from left to right along the curve  $y = x^2$  at a speed of 0.01m/sec. The bug monitors  $T(x, y)$  continuously. What is the rate of change of  $T$  as the bug passes through the point  $(1, 1)$ ?

[10] 4.

- (a) Does the function  $f(x, y) = x^2 + y^2$  have a maximum or a minimum on the curve  $xy = 1$ ? Explain.
- (b) Find all maxima and minima of  $f(x, y)$  on the curve  $xy = 1$ .

[13] **5.** Let  $G$  be the region in  $\mathbb{R}^2$  given by

$$\begin{aligned}x^2 + y^2 &\leq 1, \\0 &\leq x \leq 2y, \\y &\leq 2x.\end{aligned}$$

**(a)** Sketch the region  $G$ .

Express the integral  $\iint_G f(x, y) dA$  as

**(b)** a sum of iterated integrals  $\iint f(x, y) dx dy$ ,

**(c)** an iterated integral in polar coordinates  $(r, \theta)$  where  $x = r \cos(\theta)$  and  $y = r \sin(\theta)$ .

[13] **6.** For the integral  $I = \int_0^1 \int_{\sqrt{x}}^1 \sqrt{1+y^3} dydx.$

- (i) Sketch the region of integration.
- (ii) Evaluate  $I.$

[13] 7. A thin plate of uniform density  $k$  is bounded by the positive  $x$  and  $y$  axes and the circle  $x^2 + y^2 = 1$ . Find its centre of mass.

[13] 8. Let

$$I = \iiint_T (x^2 + y^2) dV,$$

where  $T$  is the solid region bounded below by the cone  $z = \sqrt{3x^2 + 3y^2}$  and above by the sphere  $x^2 + y^2 + z^2 = 9$ .

- (i) Express  $I$  as a triple integral in spherical coordinates.
- (ii) Express  $I$  as a triple integral in cylindrical coordinates.
- (iii) Evaluate  $I$  by any method.

