The University of British Columbia

Final Examination - April, 2010

Mathematics 200

Closed book examination Time: 2.5 hours

Last Name: ____________________, **First:** ______________ **Signature** ____________________

Student Number ____________________ **Section** ____________________

Special Instructions:

No books, notes or calculators are allowed. Use backs of sheets if extra space needed.

Rules governing examinations

1		12
2		14
3		12
4		10
5		13
6		13
7		13
8		13
Total		100

[12] **1.**(a) A surface $z(x, y)$ is defined by $zy - y + x = \ln(xyz)$.

(i) Compute $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ in terms of x, y, z.

(ii) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(x, y, z) = (-1, -2, 1/2)$.

1.(b) A surface $z = f(x, y)$ has derivatives $\frac{\partial f}{\partial x} = 3$ and $\frac{\partial f}{\partial y} = -2$ at $(x, y, z) = (1, 3, 1)$.

(i) If x increases from 1 to 1.2, and y decreases from 3 to 2.6,

find the change in z using a linear approximation.

(ii) Find the equation of the tangent plane to the surface at the point $(1, 3, 1)$.

[14] **2.** (i) For the function

$$z = f(x, y) = x^3 + 3xy + 3y^2 - 6x - 3y - 6.$$

Find and classify [as local maxima, local minima, or saddle points] all critical points of $f(x, y)$

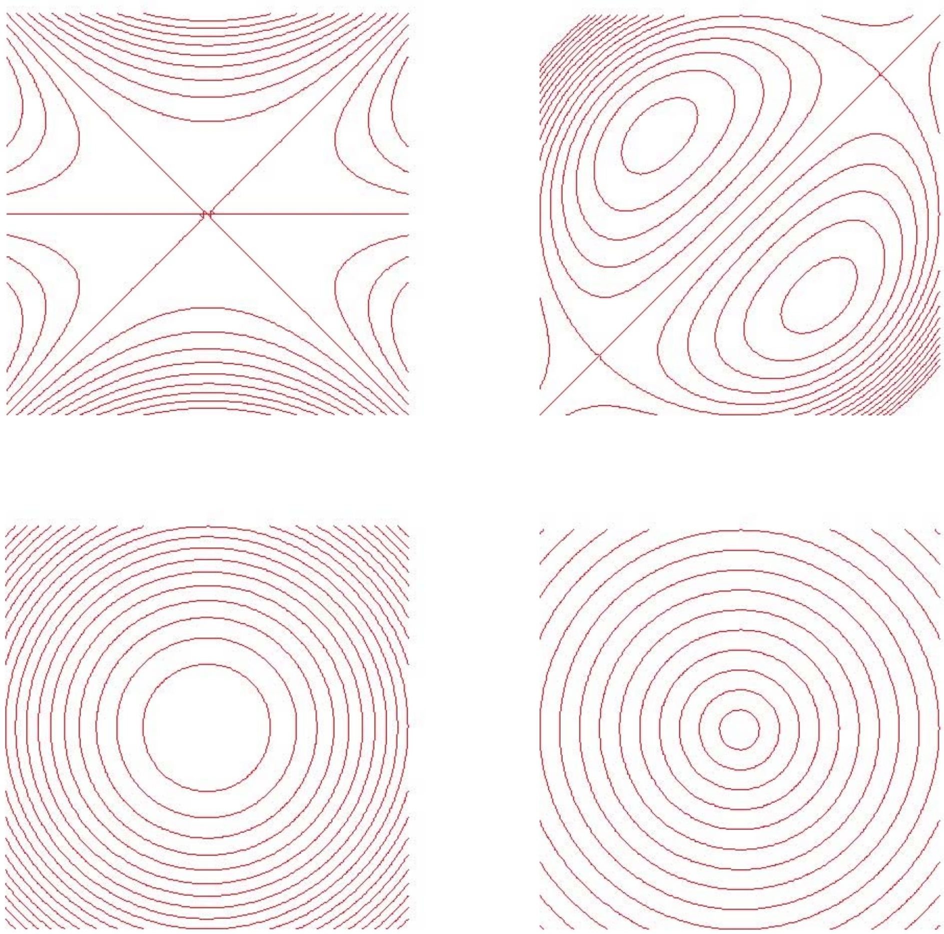
2 (ii) The images below depict level sets $f(x,y) = c$ of the functions in the list at heights $c = 0, 0.1, 0.2, \ldots, 1.9, 2$.
Label the pictures with the corresponding function and mark the critical points in each picture. (Note that in some cases, the critical points might not be drawn on the images already. In those cases you should add them to the picture.)

(a) $f(x,y) = (x^2 + y^2 - 1)(x - y) + 1$

(b) $f(x,y) = \sqrt{x^2 + y^2}$

(c) $f(x,y) = y(x + y)(x - y) + 1$

(d) $f(x,y) = x^2 + y^2$



[12] **3**. The temperature $T(x, y)$ at a point of the xy-plane is given by

$$T(x, y) = ye^{x^2}.$$

A bug travels from left to right along the curve $y = x^2$ at a speed of 0.01m/sec. The bug monitors $T(x, y)$ continuously. What is the rate of change of T as the bug passes through the point $(1, 1)$?

[10] **4.**

(a) Does the function $f(x, y) = x^2 + y^2$ have a maximum or a minimum on the curve $xy = 1$? Explain.

(b) Find all maxima and minima of $f(x, y)$ on the curve $xy = 1$.

[13] **5.** Let G be the region in $\mathbb{R}^2$ given by

$$x^2 + y^2 \leq 1,$$
$$0 \leq x \leq 2y,$$
$$y \leq 2x.$$

(a) Sketch the region G.

Express the integral $\iint_G f(x,y)dA$ as

(b) a sum of iterated integrals $\iint f(x,y)dxdy$,

(c) an iterated integral in polar coordinates (r,θ) where $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

[13] **6**. For the integral $I = \int_0^1 \int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dy dx$.

(i) Sketch the region of integration.

(ii) Evaluate I.

[13] **7**. A thin plate of uniform density k is bounded by the positive x and y axes and the circle $x^2 + y^2 = 1$. Find its centre of mass.

[13] **8**. Let

$$I = \iiint_T (x^2 + y^2)dV,$$

where T is the solid region bounded below by the cone $z = \sqrt{3x^2 + 3y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$.

(i) Express I as a triple integral in spherical coordinates.

(ii) Express I as a triple integral in cylindrical coordinates.

(iii) Evaluate I by any method.