The University of British Columbia

Final Examination - April 2007

Mathematics 200 *Calculus III*

Closed book examination **Time: 2.5 hours**

9 A.M. Section 201 - Dale Peterson 11 A.M. Section 202 - John Fournier

Last Name: ____________________, **First:** ________________ ________________

Student Number: ________________ **Signature:** ________________________

Section Number: _____

Special Instructions: No books, notes, or calculators are allowed. Show all your work. Little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than is provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBC card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		10
4		10
5		15
6		15
7		15
8		15
Total		100

[10] **1**. A plane Π passes through the points $A = (1, 1, 3)$, $B = (2, 0, 2)$ and $C = (2, 1, 0)$ in $\mathbb{R}^3$.

(a) Find an equation for the plane Π.

(b) Find the point E in the plane Π such that the line L through $D = (6, 1, 2)$ and E is perpendicular to Π.

[10] **2.** Consider the function f that maps each point (x, y) in $\mathbb{R}^2$ to ye^{-x}.

(a) Suppose that $x = 1$ and $y = e$, but errors of size 0.1 are made in measuring each of x and y. Estimate the maximum error that this could cause in $f(x, y)$.

(b) The graph of the function f sits in $\mathbb{R}^3$, and the point $(1, e, 1)$ lies on that graph. Find a nonzero vector that is perpendicular to that graph at that point.

[10] **3.** A mosquito is at the location $(3, 2, 1)$ in $\mathbb{R}^3$. She knows that the temperature T near there is given by $T = 2x^2 + y^2 - z^2$.

(a) She wishes to stay at the same temperature, but must fly in some initial direction. Find a direction in which the initial rate of change of the temperature is 0.

(b) If you and another student both get correct answers in part (a), must the directions you give be the same? Why or why not?

(c) What initial direction or directions would suit the mosquito if she wanted to cool down as fast as possible?

[10] **4**. Let F be a function on $\mathbb{R}^2$. Denote points in $\mathbb{R}^2$ by (u, v) and the corresponding partial derivatives of F by $F_u(u, v)$, $F_v(u, v)$, $F_{uu}(u, v)$, $F_{uv}(u, v)$, etc.. Assume those derivatives are all continuous. Express

$$\frac{\partial^2}{\partial x \partial y} F(x^2 - y^2, 2xy)$$

in terms of partial derivatives of the function F.
Hint: Let $u = x^2 - y^2$, and $v = 2xy$.

[15] **5.** Find all critical points for

$$f(x, y) = x(x^2 + xy + y^2 - 9).$$

Also find out which of these points give local maximum values for $f(x, y)$, which give local minimum values, and which give saddle points.

[15] **6.** Find the largest and smallest values of x^2y^2z in the part of the plane $2x+y+z=5$ where $x \geq 0$, $y \geq 0$ and $z \geq 0$. Also find all points where those extreme values occur.

[15] **7.** A region E in the xy-plane has the property that for all continuous functions f

$$\iint_E f(x,y)\,dA = \int_{x=-1}^{x=3} \left[\int_{y=x^2}^{y=2x+3} f(x,y)\,dy \right] dx.$$

(a) Compute $\displaystyle\iint_E x\,dA$.

(b) Sketch the region E.

(c) Set up $\displaystyle\iint_E x\,dA$ as an integral or sum of integrals in the opposite order.

[15] **8.** A certain solid V is a right-circular cylinder. Its base is the disk of radius 2 centred at the origin in the xy-plane. It has height 2 and density $\sqrt{x^2+y^2}$.
A smaller solid U is obtained by removing the inverted cone, whose base is the top surface of V and whose vertex is the point $(0,0,0)$.

(a) Use cylindrical coordinates to set up an integral giving the mass of U.

(b) Use spherical coordinates to set up an integral giving the mass of U.

(c) Find that mass.