Marks

[15] **1.** If two resistors of resistance $R_1$ and $R_2$ are wired in parallel, then the resulting resistance $R$ satisfies the equation $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. Use the linear or differential approximation to estimate the change in $R$ if $R_1$ decreases from 2 to 1.9 ohms and $R_2$ increases from 8 to 8.1 ohms.

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[10] **2.** Assume that the directional derivative of $w = f(x, y, z)$ at a point $P$ is a maximum in the direction of the vector $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, and the value of the directional derivative in that direction is $3\sqrt{6}$.

(a) Find the gradient vector of $w = f(x, y, z)$ at $P$. [5%]

(b) Find the directional derivative of $w = f(x, y, z)$ at $P$ in the direction of the vector $\mathbf{i} + \mathbf{j}$. [5%]

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[10] **3.** Use the Second Derivative Test to find all values of the constant $c$ for which the function $z = x^2 + cxy + y^2$ has a saddle point at $(0, 0)$.

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[15] **4.** Use the Method of Lagrange Multipliers (no credit will be given for any other method) to find the radius of the base and the height of a right circular cylinder of maximum volume which can be fit inside the unit sphere $x^2 + y^2 + z^2 = 1$.

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[10] **5.** Let $z = f(x, y)$ where $x = 2s + t$ and $y = s - t$. Find the values of the constants $a$, $b$ and $c$ such that

$$a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial^2 z}{\partial x \partial y} + c\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial s^2} + \frac{\partial^2 z}{\partial t^2}.$$

You may assume that $z = f(x, y)$ is a smooth function so that the Chain Rule and Clairaut's Theorem on the equality of the mixed partial derivatives apply.

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[10] **6.** Combine the sum of the two interated double integrals

$$\int_{y=0}^{y=1} \int_{x=0}^{x=y} f(x,y)\,dx\,dy + \int_{y=1}^{y=2} \int_{x=0}^{x=2-y} f(x,y)\,dx\,dy$$

into a single interated double integral with the order of integration reversed.

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[15] **7.** Evaluate the iterated double integral $\displaystyle\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{4-x^2}} (x^2+y^2)^{\frac{3}{2}}\, dy\, dx.$

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[15] **8.** Consider the region $E$ in 3-dimensions specified by the spherical inequalities $1 \le \rho \le 1 + \cos\varphi$.

(a) Draw a reasonably accurate picture of $E$ in 3-dimensions. Be sure to show the units on the coordinates axes. [5%]

(b) Find the volume of $E$. [10%]

**The End**

**Be sure that this examination has 9 pages**