Math 121, Spring 2010
Final Exam, April 24

Name: **SID:**

Instructor: Pramanik **Section: 201**

Problem	**Points**	**Score**
1	10	
2	20	
3	20	
4	10	
5	10	
6	15	
7	15	
8	15	
9	35	
10 (extra credit)	10	
TOTAL	150	

Instructions

- The total time is 180 minutes.
- The total score is 150 points.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- If you need extra space for your solution, use the opposite sides of the pages. Please indicate any continuing work clearly by arrows so that this work may be viewed and graded.
- There is a formula sheet on the last page of this exam booklet.
- You should not use calculators, cheat sheets or any other aids.

1. Evaluate the following indefinite integral

$$\int \frac{dx}{4\sin x - 3\cos x}.$$

(10 points)

2. This question has four parts. You can use the answer from one part in your solution for another, if you need to.

($5 \times 4 = 20$ points)

(a) Find the Maclaurin series of the function $f(x) = \ln\left(\frac{1+x}{1-x}\right)$.

(b) Find the domain of convergence of the series you obtained in part (a).

(c) Is the function $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ analytic at $x = 0$? Give reasons for your answer.

(d) For which value of n is

$$\lim_{x \to 0} x^{-n} \ln\left(\frac{1+\sin^3(x^2)}{1-\sin^3(x^2)}\right)$$

a finite nonzero constant? Find the limit in this case.

3. For each of the following series, determine whether it converges absolutely, converges conditionally or diverges. Give reasons for your answer.

(10 × 2 = 20 points)

(a)

$$\sum_{n=1}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}$$

(b)

$$\sum_{n=3}^{\infty} \frac{\cos(n\pi)}{n(\ln n)(\ln \ln n)}.$$

4. Find the area of the surface obtained by rotating the curve

$$y = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \leq x \leq 4$$

about the y axis.

(10 points)

5. A solid has a circular base of radius r. All sections of the solid perpendicular to a particular diameter of the base are squares. Find the volume of the solid.

(10 points)

6. A swimming pool 20 m long and 8 m wide has a sloping bottom so that the depth of the pool is 1 m at one end and 3 m at the other end. Find the total force exerted on the bottom if the pool is full of water.

(15 points)

7. Sketch the curves $r = 1 + \cos\theta$ and $r = 3\cos\theta$. Find the area of the region lying inside the curve $r = 1 + \cos\theta$ and outside the curve $r = 3\cos\theta$.

(5 + 10 = 15 points)

8. Let $a_1 > \sqrt{2}$ and

$$a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}.$$

Show that $\{a_n\}$ is a convergent sequence. Find the limit of this sequence.

(10 + 5 = 15 points)

9. Give brief answers to the following questions:

(7 × 5 = 35 points)

(a) Is

$$\int_0^{\frac{\pi}{2}} \csc x \, dx$$

a convergent or divergent integral? Evaluate it if it converges.

(b) Calculate $f(\sqrt{\frac{\pi}{2}})$ where

$$\frac{d}{dx}\left[x^2 \int_0^{x^2} \frac{\sin u}{u}\, du\right] - 2x \int_0^{x^2} \frac{\sin u}{u}\, du.$$

(c) Write down the form of the partial fraction decomposition of the given rational function. Do not actually evaluate the constants you use in the decomposition.

$$\frac{x^5 + x^3 + 1}{(x-1)(x^2-1)(x^3-1)}.$$

(d) Is every infinitely differentiable function analytic? Given reasons for your answer.

(e) Must every probability distribution have finite expectation? Give reasons for your answer.

(f) Solve the integral equation

$$x^2y' + y = x^2e^{\frac{1}{x}}, \qquad y(1) = 3e.$$

(g) Write down a simplified formula for the midpoint rule approximation M_n of the integral

$$\int_0^1 x^2\,dx.$$

Evaluate

$$\lim_{n\to\infty} M_n.$$

10. (Extra credit) Can a solid have finite volume and yet an infinite surface area? If yes, given an example. If not, prove why not.

(10 points)