Faculty of Mathematics

University of British Columbia

MATH 121

FINAL EXAM - Winter Term 2009

Time: 12:00-2:30 pm **Date:** April 24 , 2009.

Family Name: ____________________ First Name: ____________________

I.D. Number: ____________________ Signature: ____________________

Question	**Mark**	**Out of**
1		30
2		20
3		10
4		10
5		10
6		10
7		10
Total		100

THERE ARE 15 PAGES ON THIS TEST. THE LAST 2 PAGES ARE FOR ROUGH WORK, AND YOU MAY TEAR THEM OFF TO USE. YOU ARE NOT ALLOWED TO USE CALCULATORS, NOTES OR BOOKS TO AID YOU DURING THE TEST.

1. **Short answer questions** . Put your answers in the box provided. 3 marks will be given for correct answers in the box, while at most 1 mark will be given for incorrect answers. Unless otherwise stated, simplify your answers as much as possible.

   (a) Evaluate $\int \frac{1}{x \ln x} dx$.

   (b) For what values of α does $\int_e^\infty \frac{1}{x(\ln x)^\alpha} dx$ converge?

   (c) Find $f'(x)$ where $f(x) = \int_0^{x^2} t^2 dt$.

(d) Use the binomial series to find $f^{(6)}(0)$ where $f(x) = \frac{1}{\sqrt{1+x^3}}$ (express your answer as a fraction)

(e) Evaluate $\sum_{n=1}^{\infty} \frac{1}{3^n (n-1)!}$

(f) The waiting time to be served at a certain restaurant is assumed to be exponentially distributed according to the probability density function $\rho(t) = ce^{-ct}$ for $t \geq 0$. It is observed that 30 out of every 100 customers is served within 5 minutes of ordering. Find c.

(g) Evaluate $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{j^2}{n^3}$

(h) Estimate the error in approximating e by $\sum_{n=0}^{5} \frac{1}{n!}$

(i) Evaluate $\lim_{x\to 0} \dfrac{x^2 \sin^2 x}{(1-e^{x^2})^2}$.

(j) Find the midpoint rule approximation to $\int_1^3 \frac{1}{x} dx$ with $n = 3$.

Full solution problems In problems 2-7, justify your answers and show all your work.

2. Evaluate the following integrals

   (a) $\int \frac{x+1}{x^3+x} dx$

   (b) $\int e^{\sqrt{x}} dx$

(c) $\int \frac{x^2}{(1-x^2)^{3/2}} dx$

(d) $\int_0^{\pi/3} \frac{1}{\sin x - 1} dx$

3. (a) Let R be the region under the curve $y = \frac{1}{\sqrt{1+x^2}}$ for $0 \leq x \leq 1$. Revolve R around the x axis to obtain the solid S. Find the x coordinate $\bar{x}$ of the center of mass of S.

(b) Solve the differential equation $y' = 2xy + e^{x^2}$; $y(0) = 2$

4. Find the Taylor series along with Radii of convergence for the following

(a) $f(x) = \dfrac{1}{x}$ about $x = 2$.

(b) $f(x) = x^2 \arctan x^2$ about $x = 0$.

5. Determine if the following series are convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{e^n - n^2}$

6. Define the sequence a_n recursively by: $a_1 = 3$ and $a_{n+1} = \frac{2}{3}a_n + \frac{4}{3a_n}$.

   (a) Show that $2 \leq a_n \leq 3$ for all n.

   (b) Prove that $\{a_n\}$ converges and evaluate the limit.

7. Define the sequence $\{a_n\}$ recursively by $a_0 = a_1 = 1$ and the relation

$$a_{n+2} = \frac{2n-1}{(n+1)(n+2)} a_n.$$

(a) Show that the series $\sum_{n=0}^{\infty} a_n x^n$ converges for all x.

(b) Verify that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ solves the differential equation $f'' - 2xf' + f = 0$

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FOR ROUGH WORK ONLY...

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