Name: ____________________ Student Number: ____________________

Math 121 Final Exam April 2007 2.5 hours.

There are **11 pages** in this test including this cover page. **No calculators, books, notes, or electronic devices of any kind are permitted. Unless otherwise indicated, show all your work.**

Rules governing formal examinations:

1. Each candidate must be prepared to produce, upon request, a Library/AMS card for identification;
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions;
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination;
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action;
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
   (b) Speaking or communicating with other candidates;
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received;
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator; and
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Problem #	Value	Grade
1	36	
2	20	
3	10	
4	8	
5	9	
6	9	
7	8	
Total	100	

I have read and understood the instructions and agree to abide by them.

Signed: ____________________

1. ([36 marks]) **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, simplify your answers as much as possible.

   (a) Evaluate $\int (2y+1)^5 dy$.

   (b) Evaluate $\int_e^\infty \frac{1}{x(\ln(x))^2} dx$.

   (c) Let $f(x) = \int_{e^x}^0 \cos^3(t)dt$. Find $f'(x)$.

   (d) Calculate the volume of the solid obtained by rotating the region above the x-axis, below the curve $y = \sin(x)/x$, and between the lines $x = \pi/2$ and $x = \pi$ about the y-axis.

(e) Find the first three nonzero terms in the power series representation in powers of x (i.e. the Maclaurin series) for $\int_0^x t\cos(t^3)dt$.

(f) Evaluate $\sum_{k=2}^{\infty} 3^{k-1}2^{-2k}$.

(g) Find the general solution of the differential equation $\frac{dy}{dx} = x^2y^2$.

(h) Solve the initial value problem $\frac{dy}{dt} = (ty)^{-1}, \quad y(1) = 2$.

(i) Evaluate $\lim_{n \to \infty} \sum_{j=1}^{n} \frac{1}{n} \cos(\frac{j\pi}{2n})$.

(j) Find the midpoint rule approximation to $\int_1^3 \frac{1}{x} dx$ with $n = 3$.

(k) Evaluate $\int_{-1}^{1} \frac{1}{x^{2/3}} dx$ or show that it diverges.

(l) Evaluate $\lim_{x \to 0} \frac{1}{x^3} \int_x^{2x} \sin(t^2) dt$.

Full-Solution Problems. In questions 2-6, justify your answers and **show all your work**.

2. ([5 marks each]) Evaluate the following integrals.

(a)

$$\int \frac{x}{\sqrt{1-x^4}}dx.$$

(b)

$$\int_0^1 \frac{2x+3}{(x+1)^2}dx.$$

(c)

$$\int_0^{\pi/2} e^{-x} \cos(x) dx$$

(d)

$$\int \frac{1}{x[1 + \ln^2(x)]^{3/2}} dx.$$

3. (a) ([5 marks]) Sketch the bounded region that lies bewteen the curves $y = 4 - x^2$ and $y = (x - 2)^2$, and find its area.

(b) ([5 marks]) Let R be the region under the curve $y = e^{-x}$ and above the x-axis, for $0 \leq x \leq 1$. Find the x-coordinate of the centroid of R.

4. ([8 marks]) Find the Maclaurin series (Taylor series about $x = 0$) for the function $\tan^{-1}(x^2)$, and find its interval of convergence.

5. Determine whether each series is absolutely convergent, conditionally convergent, or divergent (and justify your answer).

   (a) ([4 marks]) $\sum_{k=1}^{\infty} \frac{2^{2k-1}}{3^k+k}$

   (b) ([5 marks]) $\sum_{k=1}^{\infty} (-1)^k \sqrt{\frac{1+1/k}{k+3}}$

6. ([9 marks]) Find all horizontal and vertical tangents of the parametric curve $x = t^3 - 3t$, $y = t^2$, and sketch it. Then find the area enclosed by the loop.

7. ([8 marks]) Let x be a number such that the power series $\sum_{j=1}^{\infty} a_j x^j$ converges.

(a) Prove that $\sum_{j=1}^{\infty} a_j (x/2)^j$ converges absolutely.

(b) Must $\sum_{j=1}^{\infty} a_j^2 x^j$ converge? (Either prove that this is the case, or else provide a counterexample.)