MATH 105 Exam Formula Sheet

- **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

- **Trigonometric formulas:**

$$\cos^2 x = \frac{1+\cos(2x)}{2}, \quad \sin^2 x = \frac{1-\cos(2x)}{2}, \quad \sin(2x) = 2\sin x \cos x$$

- **Derivatives of some inverse trigonometric functions:**

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

- **Indefinite integrals:**

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C, \qquad \int \frac{dx}{1+x^2} = \tan^{-1} x + C = \arctan x + C$$

- **Probability:**

  If X is a continuous random variable with probability density function $f(x)$ with $-\infty < x < \infty$, then the expected value $\mathbf{E}(X)$ and the variance $Var(X)$ are given by

$$\mathbf{E}(X) = \int_{-\infty}^{\infty} x f(x)\, dx, \qquad Var(X) = \int_{-\infty}^{\infty} (x - \mathbf{E}(X))^2 f(x)\, dx$$

- **Some commonly used Taylor series centered at 0:**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \qquad \text{for } |x| < 1$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \qquad \text{for } |x| < \infty$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \qquad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \qquad \text{for } |x| < \infty$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \qquad \text{for } 1 \geq |x|$$

- **Two important limits:**

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$