Marks

[42] **1.** **Short-Answer Questions**. Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Let $f(x, y) = 8x^{\frac{1}{5}}y^{\frac{4}{5}}$. Find $f(2x, 2y) - 2f(x, y)$.

Answer:

(b) Let $f(x, y) = xe^{2y} + y^2$. Evaluate $\dfrac{\partial^2 f}{\partial y^2}$.

Answer:

(c) Find all point(s) (x, y) where $f(x, y) = x^3 y - 8y - 5x$ may have a relative maximum or minimum.

Answer:

(d) Find the function $f(x)$ for which $f'(x) = \dfrac{1}{\sqrt{x}} + x^2$ and $f(1) = 2$.

Answer:

(e)   Compute the average value of $f(x) = \dfrac{1}{x}$ over the interval $1 \le x \le e$.

> Answer:

(f)   Use a Riemann sum with $n = 2$ and select the midpoints of subintervals to estimate the area under the graph of $f(x) = \dfrac{1}{1 + \sqrt{x}}$ from 0 to 4.

> Answer:

(g)   Let m be a positive number. If the volume of the solid of revolution generated by revolving about the x-axis the region under the graph of $y = \sqrt{x}$ from $x = 0$ to $x = m$ is 2π, find the positive number m.

> Answer:

(h)   Find $\displaystyle\int \dfrac{\sqrt{\ln x}}{x}\, dx$.

> Answer:

(i) Find $\int xe^{2x}\,dx$.

Answer:

(j) Use the trapezoidal rule with $n = 3$ to approximate $\int_{-2}^{4} \dfrac{dx}{1 + \left(\frac{x}{2}\right)^4}$.

Answer:

(k) Suppose that money is deposited steadily into a savings account at the rate of $3000 per year. Determine the balance at the end of 5 years if the account pays 6% interest compounded continuously.

Answer:

(l)   Let k be a constant. Find the value of k such that $f(x) = kx^{\frac{3}{2}}$ is a probability density function on $1 \le x \le 4$.

> Answer:

(m)   Compute the cumulative distribution function corresponding to the density function $f(x) = 2(x-1)$, $1 \le x \le 2$.

> Answer:

(n)   Find the variance of the random variable X whose density function is $f(x) = \dfrac{1}{2\sqrt{x}}$, $1 \le x \le 4$. (You may need the fact that the expected value of the random variable X above is $\frac{7}{3}$.)

> Answer:

Full-Solution Problems. In questions 2–6, justify your answers and show all your work.

[12]   **2.**   For a particular commodity, the demand curve is $p = D(x) = \dfrac{42}{x+2}$ and the supply curve is $p = S(x) = x + 3$. Find the consumers' surplus and producers' surplus at the sale level A, where $D(A) = S(A)$.

[12] **3.** Find the area of the shaded region in Fig. 1 bounded by $y = -\dfrac{2}{x}$, $y = -4\sqrt{x}$ and $y = -x$.

[12]   **4.**   The production function for a firm is $f(x, y) = 5x^{\frac{2}{3}}y^{\frac{1}{3}}$, where x and y are the number of units of labor and capital utilized respectively. Suppose that labor costs \$108 per unit and capital costs \$2 per unit and that the firm decides to produce 600 units of goods. Use Lagrange multipliers to determine the amounts of labor and capital that should be utilized in order to minimize the cost. You need not show that your solution minimizes the cost.

[14]   **5.**   A person purchased a home at the price \$300 000, paid a down payment equal to 20% of the purchase price, and financed the remaining balance with a 25 year term mortgage. Assume that the person makes payments continuously at a constant annual rate A and that interest is compounded continuously at the rate of 5%.

(a)   Write down the differential equation that is satisfied by the amount $y(t)$ of money owed on the mortgage at time t.

(b)   Determine A, the rate of annual payments, that is required to pay off the loan in 25 years.

(c)   Determine the total interest paid during the 25 year term mortgage.

[8]   **6.**   Let

$$f(x) = \lim_{n \to \infty} \left[\left(\frac{1}{3} + \frac{1}{2 + \left(1 + \frac{x-1}{n}\right)^3} + \frac{1}{2 + \left(1 + \frac{2(x-1)}{n}\right)^3} + \ldots + \frac{1}{2 + \left(1 + \frac{(n-1)(x-1)}{n}\right)^3} \right) \frac{x-1}{n} \right]$$

where $x \geq 0$. Find the equation of the tangent line to the graph $y = f(x)$ at $x = 1$.

The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia
Final Examination - April 24, 2010

Mathematics 105
All Sections

Closed book examination   Time: 2.5 hours

Name ——————————————   **Signature** ——————————————

Student Number——————————   **Instructor's Name** ——————————

Section Number ——————————

Special Instructions:

No memory aids are allowed. One Sharp EL-510R calculator, WITH COVER REMOVED, may be used. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1		42
2		12
3		12
4		12
5		14
6		8
Total		100