Marks

[42] **1. Short-Answer Questions**. Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Assume that $z(x, y)$ is a linear function with slope 2 in the x-direction and slope -3 in the y-direction. If $z(1, 1) = 4$, find $z(-2, 1)$.

Answer:

(b) If $f(x, y) = \ln(x^2 + y)$, find $\lim_{k \to 0} \dfrac{f(1 + k, 0) - f(1, 0)}{k}$.

Answer:

(c) Let (x_0, y_0) be a critical point of $f(x, y) = -x^2 - y^2 + 6x + 8y - 21$. Find (x_0, y_0) and then find the equation of the tangent plane to the surface $f(x, y)$ at the point $\big(x_0, y_0, f(x_0, y_0)\big)$.

Answer:

(d) Suppose the marginal revenue in producing x units of a certain product is $MR(x) = 300 - 0.2x$. Find the change in total revenue if production is increased from 10 to 20 units.

Answer:

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(e) Find $\int \frac{1+x}{x-x^2}\,dx$.

Answer:

(f) If $\int_0^1 f(x)\,dx = 2$ and $f(1) = 3$, find $\int_0^1 5xf'(x)\,dx$.

Answer:

(g) You are given the following table of values for $f(x)$:

x	1.5	2.0	2.5	3
$f(x)$	−0.6	0.2	0.4	0.8

Estimate $\int_{1.5}^{3} f(x)\,dx$ by using the trapezoidal rule with $n = 3$.

Answer:

(h) Let $p = S(q) = 10(e^{0.02q} - 1)$ be a supply curve, where p denotes the price, and q denotes the quantity supplied. Find the average price over the supply interval $[20, 30]$.

Answer:

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(i) Determine whether $\int_1^\infty \frac{dx}{\sqrt{x}}$ converges or diverges.

Answer:

(j) Find $\int_0^1 t \sin(\pi t^2)\, dt$.

Answer:

(k) If k is a nonzero constant and $y = -\frac{k}{t^3}$ is a solution of the differential equation $\frac{dy}{dt} = 6t^2y^2$, find k.

Answer:

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(l) Find the constant c such that the function

$$f(x) = cx^2(1-x), \quad 0 \le x \le 1$$

is a probability density function.

Answer:

(m) Let X be a continuous random variable having the probability density function $f(x) = \dfrac{3}{x^4}, x \ge 1$. Find the expected value $E(X)$.

Answer:

(n) Let $f(x)$ be the probability density function of a continuous random variable X, where $1 \le x \le 5$. If the area under the graph of $y = f(x)$ from $x = 3$ to $x = 5$ is $\dfrac{1}{3}$, find the probability $P(1 \le X \le 3)$.

Answer:

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Full-Solution Problems. In questions 2–6, justify your answers and show all your work.

[10] **2.** Find the total area of all the regions completely enclosed by the graphs of the functions $f(x) = x^3 - 3x + 4$ and $g(x) = x + 4$.

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[12] **3.** For a certain item the demand curve is $p = D(q) = -0.2q^2 + 60$, and the supply curve is $p = S(q) = 0.1q^2 + q + 20$. Find the consumer surplus.

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[12] **4.** Suppose that $\$100,000$ is deposited in an account paying 5% interest with continuous compounding. Also assume that money is continuously withdrawn from the account at a rate of $\$10,000$ per year. Find the amount of money in the account at the end of 6 years.

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[12] **5.** An open rectangular box without a top is to be constructed from material that costs \$5 per square foot for the bottom and \$2 per square foot for its sides. The bottom of the rectangular box is a square. Use the method of Lagrange multipliers (no credit will be given for any other method) to find the dimensions of the box of greatest volume that can be constructed for \$240. You do not need to show that the answer you compute gives the greatest volume.

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[12] **6.** Suppose that money is deposited continuously into a savings account at a rate of $200t$ dollars per year for 10 years. No money is withdrawn during the 10 year period. The savings account earns 10% interest, compounded continuously.

(a) Find the amount of money in the account at the end of 10 years.

(b) Find the total amount of interest earned in dollars by the savings account over the 10 year period.

The End

Be sure that this examination has 10 pages including this cov