The University of British Columbia

Final Examination - December 6, 2014

Mathematics 104/184

All Sections

Closed book examination — Time: 2.5 hours

Last Name ____________ **First** ____________ **Signature** ____________

MATH 104 or MATH 184 (Circle one) **Student Number** ____________

Instructor's Name ____________ **Section Number** ____________

Special Instructions:

No books, notes, or calculators are allowed.

1		42
2		9
3		15
4		10
5		17
6		7
Total		100

[42] **1. Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\lim_{x \to 3} \dfrac{x^2 - 9}{3 - x}$.

Answer:

(b) Evaluate $\lim_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16}$.

Answer:

(c) Let $f(x) = \begin{cases} 3x & \text{if} \quad x \leq 2 \\ x^2 - 2x & \text{if} \quad x > 2 \end{cases}$. Is f continuous from the right at 2?

Answer:

(d) Let $f(x) = \dfrac{x^3}{3} + x^2 - 7x + 100$. Find the point on the graph of f at which the tangent line has slope -8.

Answer:

(e) Suppose $f(4) = 3$ and $f'(4) = 2$. Let $h(x) = \dfrac{f(x)}{x-5}$. Find an equation of the line tangent to $y = h(x)$ at $x = 4$.

Answer:

(f) Find $\dfrac{dy}{dx}$, where $y = e^{9x}\cos x$.

Answer:

(g) Suppose the position of an object moving horizontally after t seconds is given by the function $s = f(t) = t^4 - 2t$ for $0 \leq t \leq 6$. Find the acceleration of the object at $t = 2$.

Answer:

(h) Assume f and g are differentiable on their domains with $h(x) = f\big(g(x)\big)$. Suppose the equation of the line tangent to the graph of g at the point $(4, 7)$ is $y = 3x - 5$ and the equation of the line tangent to the graph of f at $(7, 9)$ is $y = -2x + 23$. Find $h'(4)$.

Answer:

(i) Find $\dfrac{dy}{dx}$, where $y = x^{\ln x}$.

Answer:

(j) An investment earns at an annual interest rate of 5% compounded continuously. How fast is the investment growing when its value is \$500?

Answer:

(k) Find $f'(0)$, where $f(x) = \tan^{-1}(5x)$. Note that $\tan^{-1}$ refers to the inverse tangent function, which is also denoted by arctan.

Answer:

(l) Suppose f is differentiable on $(-\infty, \infty)$ and assume it has a local extreme value at the point $x = 2$, where $f(2) = 0$. Let $h(x) = xf(x) + 2x + 3$ for all values of x. Does $h(x)$ have a local extreme value at $x = 2$?

Answer:

(m) Estimate $\left|\sin(0.12) - 0.12\right|$ by considering the linear approximation of $\sin x$ at $x = 0$.

Answer:

(n) Let $f(x) = x^{2/3}$. Find the second-order Taylor polynomial for $f(x)$ with its center at 1.

Answer:

Long Problems. In questions 2–6, show your work. No credit will be given for the answer without the correct accompanying work.

[9] **2**. Let $f(x) = \dfrac{x^2}{x^2+1}$. Use the definition of the derivative to find $f'(1)$. No marks will be given for the use of any differentiation rules.

[15] **3**. The price p (in dollars) and the demand q for a product are related by the following demand equation:

$$p^3 + q + q^3 = 38.$$

(a) Find the price elasticity of demand in terms of p and q for this product.

(b) If the current demand is $q = 3$, will the revenue increase or decrease if the price is raised slightly?

c) Suppose the price increases at a rate \$7/month. How fast does the demand decrease when the demand is $q = 3$?

[10] **4**. You are planning a city tour for a group of 100 tourists. If you can sell x bus tour tickets, you can offer them for \$$(30 - x/4)$ each. If you can sell y boat tour tickets, you can offer them for \$$(70 - y/2)$ each. How many bus tickets, and how many train tickets should you sell to the tourists in order to maximize revenue (you can only sell one type of ticket to each passenger).

[17] **5**. Let $f(x) = \dfrac{e^x}{x^2}$.

(a) Find the critical point of $f(x)$.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find $f''(x)$.

(d) Find the intervals on which f is concave upward.

(e) Find the horizontal asymptote and the vertical asymptote of $f(x)$. Note that $\lim_{x\to\infty} \dfrac{e^x}{x} = \infty$.

(f) Sketch the graph of $f(x) = \dfrac{e^x}{x^2}$.

[7] **6**. Let $f(x) = \dfrac{x^4}{12} - \dfrac{4(3^{-x})}{(\ln 3)^2} + \log_{(x+1)^2}(x+1)^{2014}$ be a function defined on the interval $(0, \infty)$. Show that the graph of $f(x)$ on the interval $(0, \infty)$ has an inflection point.

The End