The University of British Columbia

Final Examination - December 5, 2012

Mathematics 104/184

Time: 2.5 hours

LAST Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

First Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   **Signature** ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Student Number ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

MATH 104 or MATH 184 (Circle one)   **Section Number:**⎯⎯⎯⎯⎯⎯

Special Instructions: No memory aids are allowed. No communication devices allowed. No calculators allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Question	Points	Score
1	45	
2	13	
3	10	
4	10	
5	10	
6	12	
Total:	100	

1. (45 points) **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work, except for multiple choice questions.

   (a) Compute $\lim\limits_{x \to 0} \dfrac{\sqrt{x+1}-1}{x}$.

   Answer:

   (b) For what value of the constant a is the following function continuous at $x = 1$?

   $$f(x) = \begin{cases} \dfrac{xe^x - e^x}{x^2 - 3x + 2} & \text{if } x \neq 1, \\ a & \text{if } x = 1. \end{cases}$$

   Answer:

   (c) Differentiate the function

   $$f(x) = (x+1)^{\cos x}$$

   .

   Answer:

(d) Let $h(x) = e^{f(x)} + [f(x)]^2$ where $f(1) = 2$, $f'(1) = 5$. Find $h'(1)$.

Answer:

(e) Sketch the graph of a continuous function $f(x)$ satisfying:

$$\lim_{x \to 0^-} f'(x) = 1, \ \lim_{x \to 0^+} f'(x) = -1, f(0) = 3$$

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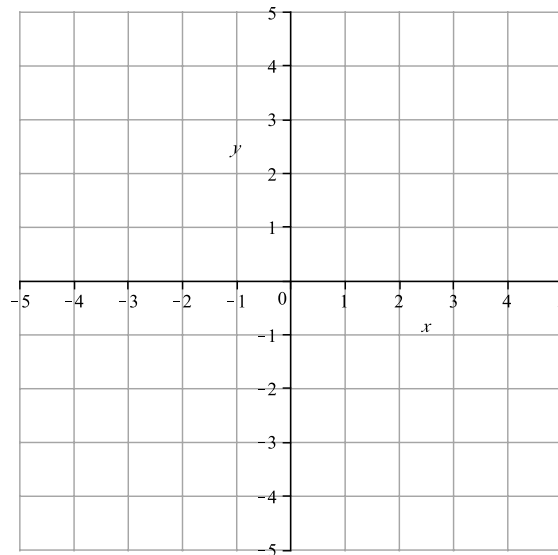
(f) Let $y(x) = e^{rx}$. For what value of r does $y(x)$ satisfy $y(x)'' - 4y(x)' + 4y(x) = 0$?

Answer:

(g) Find the second order Taylor polynomial for $y = \arctan x$ at $x = 1$.

Answer:

(h) Suppose you approximate $\cos(.25)$ by $L(.25)$, where $L(x)$ is the linear approximation of $\cos x$ at $x = 0$. Provide an estimate for the absolute value of the error in your approximation.

Answer:

(i) Let $p(q) = e^{3q}$. Consider its 20th order Taylor polynomial at $q = 1$;

$$T_{20}(q) = a_0 + a_1(q - 1) + a_2(q - 1)^2 + \ldots + a_{20}(q - 1)^{20}$$

What is a_{20}?

Answer:

(j) You invest $ 100,000$ now at an annual interest rate of 7%, compounded continuously. Your plan is to retire once the rate of growth of your investment is $ 10,000$ per year. In how many years will you retire?

Answer:

(k) The quadratic approximation of a demand function $q(p)$ at $p = 10$ is

$$T_2(p) = 2 - 4(p - 10) + 3(p - 10)^2$$

Calculate $\varepsilon(10)$ where $\varepsilon(p)$ is the price-elasticity of demand.

Answer:

(l) If $f'(a)$ exists, then $\lim\limits_{x \to a} f(x)$

   (i) equals $f(a)$.
   (ii) equals $f'(a)$.
   (iii) must exist, but may not equal $f(a)$ or $f'(a)$.
   (iv) might not exist.
   (v) None of the above.

Answer:

(m) On what intervals is the function $f(t) = t^2 e^t$ increasing? Decreasing?

The intervals of increase are:

The intervals of decrease are:

(n) Find the derivative of the function $f(x) = \ln(1 + \ln(1 + \ln x))$

Answer:

(o) Show that the equation $5^x - 10x - 7 = 0$ has a solution.

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

2. Consider the function
$$f(x) = \frac{2x^3}{3x^2 - 9}.$$

   Its first and second derivatives are given by
$$f'(x) = \frac{2x^2(x^2 - 9)}{3(x^2 - 3)^2}, \qquad f''(x) = \frac{4x(x^2 + 9)}{(x^2 - 3)^3}.$$

   (a) (4 points) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

The intervals of increase are:

The intervals of decrease are:

   (b) (3 points) On which intervals is $f(x)$ concave up? On which intervals is $f(x)$ concave down?

Concave up on these intervals:

Concave down on these intervals:

   (c) (2 points) Find the x coordinate of all local maxima, local minima, and inflection points. Be sure to indicate which is which.

(d) (3 points) Write the equation of all vertical asymptotes (if there are any). Show that $y = \dfrac{2}{3}x$ is a slant asymptote.

(e) (1 point) In the box provided, indicate which curve below best represents the graph of $y = f(x)$. You will get 0 points for not making a choice. You will get -1 points for making the wrong choice.

Answer:

a>

b>

c>

d>

e>

f>

g>

h>

i>

j>

k>

l>
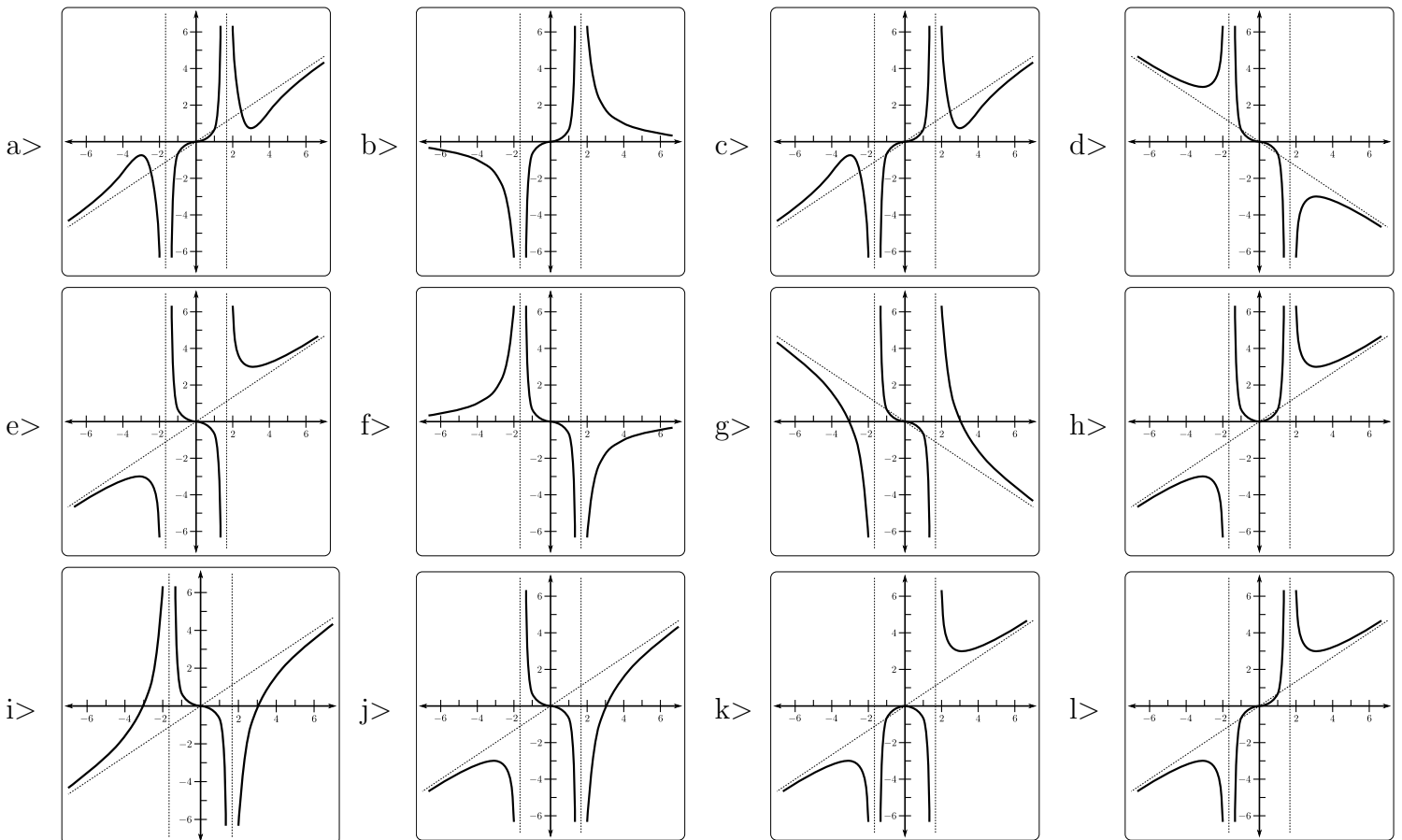
3. (10 points) A steel company, ABC steel, manufactures nuts and bolts. When x nuts are produced, they can be sold for $-3x + 500$ dollars each. When y bolts are produced, they can be sold for $-y + 300$ dollars each.

Assume that nuts and bolts weigh 0.5 kg each. How many nuts and how many bolts must be produced to maximize the revenue from 100 kg of steel? Justify your answer.

4. The demand equation for a certain product is

$$q^2 + p^{3/2} + 2p = 20$$

where q is the number of units per hour the manufacturer can sell at a price of p dollars per unit.

(a) (4 points) Calculate q and $\frac{dq}{dp}$ when the price is set at $4.

(b) (2 points) If the price is raised slightly from $4 dollars, will the revenue increase or decrease (use the elasiticity of demand to do this)?

(c) (4 points) At what rate (in dollars per hour) must the price above ($ 4) be changed at this instant so that the revenue changes at 0.15 dollars per hour?

5. Alice is 5 ft tall and Bob is 6 ft tall. At some instant they are both walking towards a street lamp 15 ft high with Alice west of the lamp and Bob east of the lamp. Suppose at this instant, Alice is walking at 4 ft/s and the distance between the tips of their shadows is decreasing at 11 ft/s.

(a) (2 points) Draw a clear diagram for the situation. Let x be the distance between Alice and the lamp, and let y the distance between Bob and the lamp (you may label other distances in your diagram as well).

(b) (8 points) How fast is Bob walking?

6. Consider the curve $x^2 + y^3 - 2xy = 0$. Assume that the point $(x, y) = (1, 1)$ lies on the curve, and that nearby points on the curve satisfy $y = f(x)$ for some function of $f(x)$.

   (a) (3 points) Find $f(1)$ and $f'(1)$.

   (b) (3 points) Find $f''(1)$.

(c) (3 points) Approximate $f(1.02)$ using the linear approximation of $f(x)$ at $x = 1$. State whether you expect this to be an overestimate, or an underestimate of $f(1.02)$.

(d) (3 points) Write down the quadratic approximation of $f(x)$ at $x = 1$. Sketch its graph in the x, y grid below.
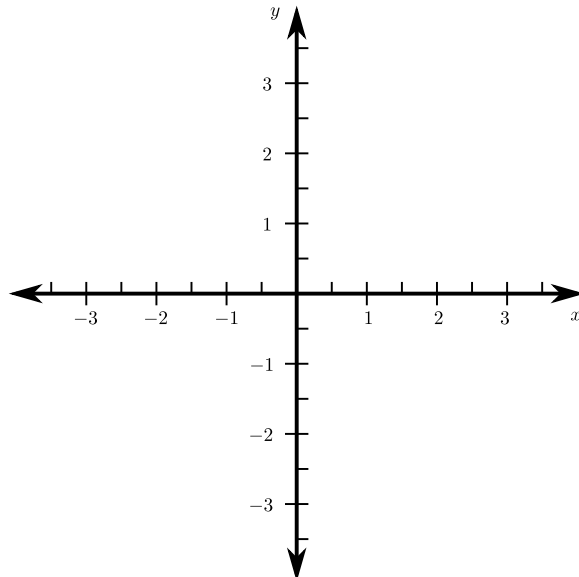
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