The University of British Columbia

Final Examination - December 11, 2013

Mathematics 104/184

Time: 2.5 hours

LAST Name ______________________________

First Name ____________________ **Signature** ____________________

Student Number ________________

MATH 104 or MATH 184 (Circle one) **Section Number:**__________

Special Instructions:

No memory aids are allowed. No communication devices allowed. No calculators allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		42
2		14
3		10
4		12
5		10
6		12
Total		100

[42] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work, except for multiple choice questions.

(a) Find $f'(x)$ where $f(x) = \dfrac{\sin(x)}{e^x + 7}$. Do NOT simplify your answer.

Answer:

(b) Find the value of a for which $f(x)$ is continuous at $x = -1$, where

$$f(x) = \begin{cases} \dfrac{x^2 + 3x + 2}{x + 1} & \text{if } x \neq -1, \\ a & \text{if } x = -1. \end{cases}$$

Answer:

(c) Compute $\displaystyle\lim_{x\to\infty} \frac{9x^2 - 6x + 8}{-3x^3 + 4}$.

Answer:

(d) Find the slope of the tangent line to $y = \ln(x^2)$ at $x = e^2$.

Answer:

(e) Find the slope of the tangent line to $y = x^3 + 3x^2 + 2$ at its point of inflection.

Answer:

(f) If $y = xy + x^2 + 1$, find the equation of the tangent line to this curve at the point $(-1, 1)$.

Answer:

(g) Find the interval(s) on which $f(x) = x + \dfrac{1}{x}$ is increasing.

Answer:

(h) You borrow 10 thousand dollars from Nick the Shark, who charges you at a fixed rate r that is compounded continuously. If you pay Nick 100 thousand dollars 2 years later, what was the annual rate of interest that he charged? (A calculator-ready form will suffice.)

Answer:

(i) Find an approximation to $\ln(1.25)$ by using the linear approximation to $f(x) = \ln(x)$ at $a = 1$.

Answer:

(j) If a function f is continuous for all x and if f has a local maximum at $(-1, 4)$ and a local minimum at $(3, -2)$, which of the following statements *must* be true?

(A) The graph of f has an inflection point somewhere between $x = -1$ and $x = 3$.

(B) $f'(-1) = 0$.

(C) The graph of f has a horizontal asymptote.

(D) The graph of f has a horizontal tangent line at $x = 3$.

(E) The graph of f intersects both axes.

Answer:

(k) At $x = 0$, which of the following is true for the function $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.

(B) f is decreasing.

(C) f is discontinuous.

(D) f has a local minimum.

(E) f has a local maximum.

Answer:

(l) If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$.

(E) None of the above.

Answer:

(m) Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $a = 3$ is used to approximate $f(x)$, then an approximate solution for x to the equation $f(x) = 0$ is

(A) $x = 0.4$

(B) $x = 0.5$

(C) $x = 2.6$

(D) $x = 3.4$

(E) $x = 5.5$

Answer:

(n) If the base b of a triangle is increasing at a rate of 3 centimetres per second while its height h is decreasing at a rate of 3 centimetres per second, which of the following must be true about the area A of the triangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is decreasing only when $b < h$.

(D) A is decreasing only when $b > h$.

(E) A remains constant.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[14] **2**. Consider the function

$$f(x) = \frac{x^2}{x^2 - 4}.$$

Its first and second derivatives are given by

$$f'(x) = -\frac{8x}{(x^2-4)^2}, \qquad f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}.$$

(a) Find all x such that $f'(x) = 0$ or $f'(x)$ does not exist.

(b) Find all x such that $f''(x) = 0$ or $f''(x)$ does not exist.

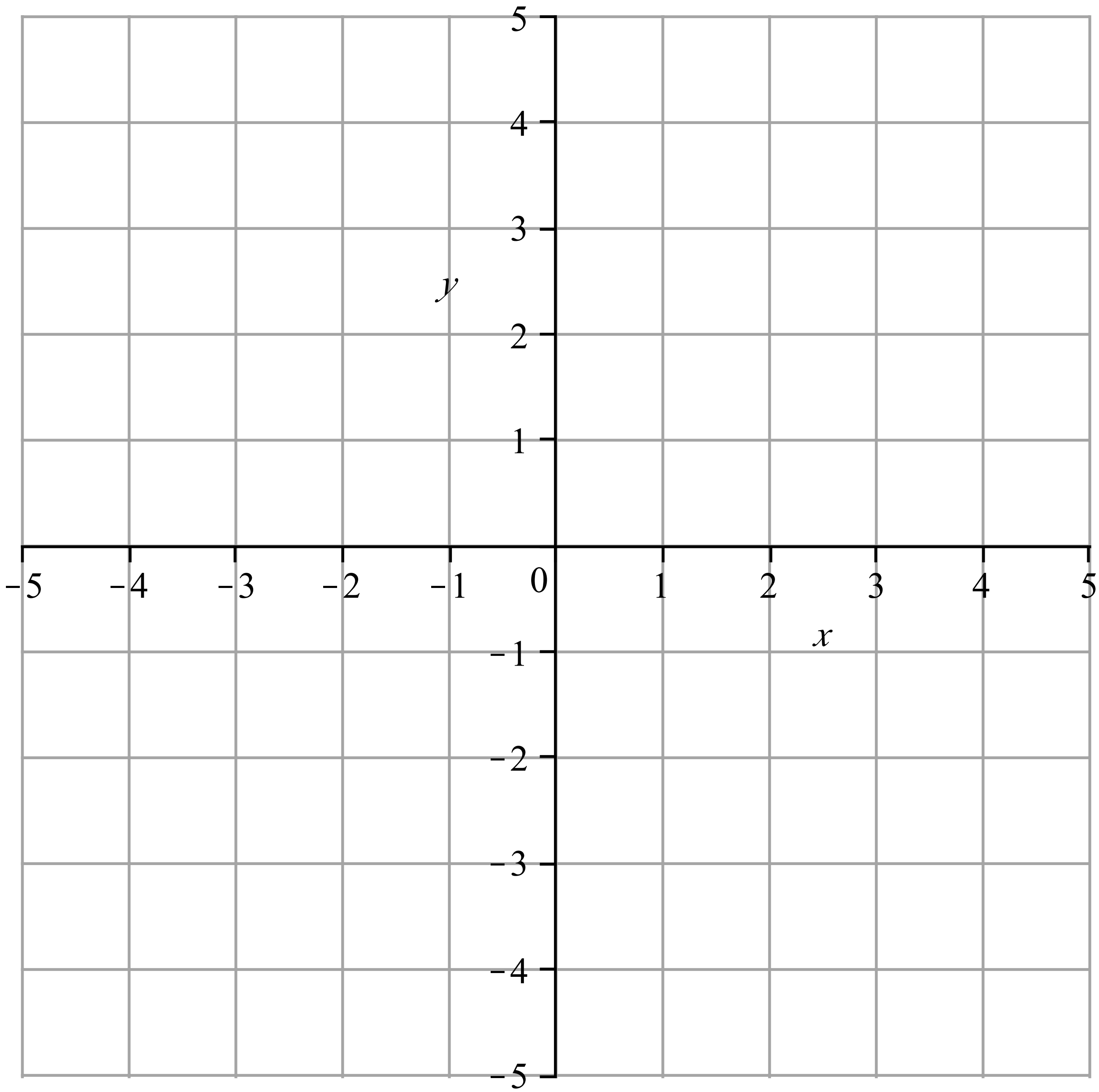
(c) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

(d) On which intervals is $f(x)$ concave up? On which intervals is $f(x)$ concave down?

(e) Find the coordinates of all local maxima, local minima, and inflection points. Be sure to indicate which is which.

(f) Find any horizontal and vertical asymptotes of the function $f(x)$ and write their equations.

(g) Draw the graph of $f(x)$ on the graph provided. Accurately place all critical points and inflection points, indicate all asymptotes, and make sure your graph correctly shows where $f(x)$ is increasing and decreasing and correctly shows its concavity.



[10] **3**. Two cylindrical tanks are being filled simultaneously at exactly the same rate. The smaller tank has a radius of 5 metres, and the water rises at a rate of 0.5 metres per minute. The larger tank has a radius of 8 metres. How fast is the water rising in the larger tank? It may be helpful to know that the volume of a cylinder of radius R and height H is $V = \pi R^2 H$.

Answer:

[12] **4**. A rectangular storage container with an open top is to have volume of 8 cubic metres. The length of its base is twice the width. Material for the base costs \$4.50 per square metre, and material for the sides costs \$6 per square metre. Find the cost of the material for the cheapest such container.

Answer:

[10] **5**. Currently 1800 people ride a commuter passenger ferry each day and pay $4 for a ticket. The number of people q willing to ride the ferry at price p is determined by the relationship

$$p = \left(\frac{q - 3000}{600}\right)^2.$$

The company would like to increase its revenue. Use the price elasticity of demand ϵ to give advice to management on whether it should increase or decrease its price from $4 per passenger. Recall that $\epsilon = \frac{p}{q}\frac{dq}{dp}$.

[12] **6**. (a) Use the linear approximation to $f(x) = \sqrt[3]{x} = x^{1/3}$ at $a = 8$ to approximate $\sqrt[3]{7}$.

(b) Use the formula $|\text{Error}| \leq \dfrac{M}{2}(x-a)^2$, where $M > 0$, to estimate an error bound for your approximation of $\sqrt[3]{7}$ in part (a). It will be useful to remember how to find such an M.

(c) Is your approximation for $\sqrt[3]{7}$ too large or too small? Explain. Use this information together with the error bound you found in part (b) to construct the smallest interval you can guarantee contains the true value of $\sqrt[3]{7}$.