The University of British Columbia

Final Examination - December 13, 2008

Mathematics 104/184

All Sections

Closed book examination Time: 2.5 hours

Last Name ____________ **First** ________ **Signature** ____________

Student Number ____________

MATH 104 or MATH 184 (Circle one) **Section Number:**________

Special Instructions:

No memory aids are allowed. One Sharp EL-510R calculator, WITH COVER REMOVED, may be used. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		42
2		12
3		12
4		12
5		12
6		10
Total		100

[42] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Find the number c that makes

$$f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{x^2 - x - 6} & \text{if } x \neq 3 \\ c & \text{if } x = 3 \end{cases}$$

continuous for every x in its domain.

Answer:

(b) Evaluate $\displaystyle\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$.

Answer:

(c) Evaluate $\displaystyle\lim_{x \to \infty} \frac{x^3 - 5x + 17}{7x^3 + 5x^2 + x}$.

Answer:

(d) Find the marginal revenue at a production level of 20 units if the revenue from producing and selling q units of a product is given by $R(q) = 3q - 0.1q^2 + \ln(0.5q)$ dollars.

Answer:

(e) The point with $x = e$ is a critical point of the function $f(x) = \dfrac{\ln x}{x}$. Is $x = e$ a relative maximum or a relative minimum point for f?

Answer:

(f) Find the amount of money (to the nearest dollar) that needs to be invested today at an interest rate of 4% compounded continuously in order that it grow to \$10 000 in 7 years.

Answer:

(g) Find the interval where $f(x) = xe^{-x}$ is decreasing.

Answer:

(h) Find the minimum value of the function $f(x) = x^3 - 3x^2 - 9x + 2$ on the interval $-2 \le x \le 2$.

Answer:

(i) Find the equation of the tangent line to $y = \tan^{-1}(x^2)$ at the point $\left(1, \frac{\pi}{4}\right)$. (Note: Another notation for $\tan^{-1}(x)$ is $\arctan(x)$.)

Answer:

(j) Find the values of a and b so that

$$f(x) = \begin{cases} ax + b & \text{if } x < 0 \\ 2\sin x + 3\cos x & \text{if } x \geq 0 \end{cases}$$

is differentiable at $x = 0$.

Answer:

(k) The value of a house grows at a continuous rate of $r\%$ per year. Find the growth rate such that the value of the house doubles in 3 years.

Answer:

(l) Determine the second degree Taylor polynomial of $f(x) = \sqrt{x}$ at $x = 9$.

Answer:

(m) Find the equation of the tangent line to $x^2y^4 = 1$ at the point $(4, \frac{1}{2})$.

Answer:

(n) The price p (in dollars) and the demand q (in thousands of units) of a commodity satisfy the demand equation $6p + q + qp = 94$. Find the rate at which demand is changing when $p = 9$, $q = 4$, and the price is rising at the rate of \$2 per week.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[12] **2**. Let $y = f(x) = \dfrac{x^2}{x^2+1}$.

In this question, you may use without verifying it that $f''(x) = \dfrac{2(1-3x^2)}{(1+x^2)^3}$.

(a) Find the intervals on which $f(x)$ is increasing and on which it is decreasing. [3pts]

(b) Find the intervals on which $f(x)$ is concave upward and on which it is concave downward, as well as the x-coordinates of any inflection points. [3pts]

(c) Find any asymptotes for $y = f(x)$. [2pts]

(d) Sketch the graph of $y = f(x)$. Identify on your graph any critical points, singular points, local maxima and local minima, and inflection points. Also, indicate any asymptotes that exist. [4pts]

[12] **3**. The monthly advertising revenue A and the monthly circulation x of a magazine are related by the equation

$$A = 6\sqrt{x^2 - 400}, \quad x \geq 20,$$

where A is given in thousands of dollars and x is measured in thousands of copies sold. At what rate is the advertising revenue changing if $x = 25$ thousand copies, and the circulation is changing at the rate of 2 thousand copies per month?

Answer:

[12] **4**. A cell phone supplier has determined that demand for its newest cell phone model is given by

$$qp + 30p + 50q = 8500,$$

where q is the number of cell phones the supplier can sell at a price of p dollars per phone. You may find it useful in this problem to know that elasticity of demand is defined to be $E(p) = -pf'(p)/f(p)$ for the demand function $q = f(p)$.

(a) If the current price is \$150 per phone, will revenue increase or decrease if the price is lowered slightly?

Answer:

(b) What price should the cell phone supplier set for this cell phone to maximize its revenue from sales of the phone?

Answer:

[12] **5**. A furniture store expects to sell 640 sofas at a steady rate next year. The manager of the store plans to order these sofas from the manufacturer by placing several orders of the same size spaced equally throughout the year. The ordering cost for each delivery is \$160, and carrying costs, based on the average number of sofas in inventory, amount to \$32 per year for one sofa. Determine how many sofas the manager should request each time she places an order to minimize the inventory cost (which is the sum of the ordering costs and the carrying costs).

Answer:

[10] **6**. A \$1000 television is purchased with a loan to be repaid in 5 monthly installments of \$220. Use two iterations of the Newton-Raphson method, starting with an initial guess of $i_0 = 0.03$, to approximate the monthly rate of interest on the loan. You may find it useful to know the formula $Pi + R[(1+i)^{-N} - 1] = 0$.

Answer:

The End