**Be sure this exam has 9 pages including the cover**

**The University of British Columbia**

**MATH 103**

**Final Exam – Apr 11, 2012**

**Family Name** ______________________ **Given Name** ______________________

**Student Number** ______________________ **Signature** ______________________

**Section Number** ______________________

This exam consists of **5** questions. No notes nor calculators. Note the number of marks for each question. Use your time wisely.

| Problem | max score | score |
|---|---|---|
| 1. | 14 | |
| 2. | 18 | |
| 3. | 6 | |
| 4. | 14 | |
| 5. | 6 | |
| total | 58 | |

(14 points) 1. Answer the following multiple choice questions. Check your answer very carefully. Your answer will be marked right or wrong (work will not be considered for this problem).

(2 points) (a) Does the sum $\sum_{n=1}^{\infty} \frac{n^{0.1}}{n^{0.99}+n^{1.1}+1}$ converge?

A. Converge.

B. Diverge.

(a) ______________

(2 points) (b) Does the integral $\int_1^{\infty} \frac{x^2}{x^3+2x+5} dx$ converge?

A. Converge.

B. Diverge.

(b) ______________

(2 points) (c) Does the integral $\int_1^{\infty} \frac{1+\sin x}{x^2} dx$ converge?

A. Converge.

B. Diverge.

(c) ______________

(2 points) (d) The value of the integral $\int_0^{\pi} \sin^3 x dx$ is

A. 0

B. 1

C. $\pi/2$

D. $\pi$

E. $4/3$

(d) ______________

(2 points) (e) The value of the integral $\int_0^\infty x^2 e^{-x} dx$ is

A. $e - 1$
B. $2e - 1$
C. $0.5e$
D. 1
E. 2

(e) ______________

(2 points) (f) In the Taylor series of $x \log(1 - x)$ (about the point $x = 0$), the coefficient of the $x^3$ term is

A. $1/3$
B. $-1/3$
C. $1/2$
D. $-1/2$

(f) ______________

(2 points) (g) Which of the following is closest approximation to the value of the integral $\int_0^1 \frac{1 - \cos x}{x} dx$?

(Hint: use Taylor series)

A. 1
B. $1/4$
C. $1/2$
D. $5/6$

(g) ______________

(18 points) 2. The following are short answer questions. Unless otherwise indicated, simplify your answer to the best possible. Show in detail how you arrive at your answer (work will be considered for this problem).

(3 points) (a) Use the integral comparison test to show that

$$\int_1^\infty \frac{e^{-x}}{2x-1}\,dx$$

converges.

(3 points) (b) Use the formula arc length to find an integral of the form $\int_\alpha^\beta h(x)\,dx$, with suitable $\alpha$, $\beta$, and $h(x)$, that represents the circumference of the ellipse

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1,$$

where $a$ and $b$ are positive constants. **Do not evaluate this integral!**

(3 points) (c) Find the center of mass of a distribution

$$\rho(x) = \cos\left(\frac{x}{2}\right), \quad 0 < x < \pi.$$

(c) ________________

(3 points) (d) The value of the integral $\int_2^4 \frac{2}{3x^2 - 4x + 1}\, dx$ is

(d) ______________

(3 points) (e) In (a simplified version of) the dice game *Pass the Pigs* two model pigs serve as dice. Each model pig can either land on the side (with probability $p(S) = 4/8$), on the feet (with probability $p(F) = 3/8$), or on the snout (with probability $p(N) = 1/8$). In each turn both model pigs are rolled. The scoring is as follows:

| | pig 1 side | pig 1 feet | pig 1 snout |
|---|---|---|---|
| pig 2 side | 0 | 1 | 5 |
| pig 2 feet | 1 | 3 | 7 |
| pig 2 snout | 5 | 7 | 10 |

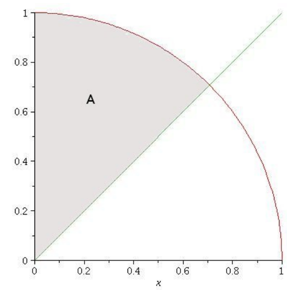
Find the average score per turn. **You can leave your answer unsimplified as a sum of fractions.**

(e) ______________

(3 points) (f) Show that $y(x) = \frac{\ln(x)+2}{x}$ is a solution to the differential equation

$$x^2y' + xy = 1.$$

(6 points) 3. Consider the area $A$ between a circle of radius 1 in the first quadrant and $f(x) = x$.




(3 points) (a) Find the volume of the solid obtained by rotating $A$ about the $x$-axis.

(a) ________________

(3 points) (b) Find the volume of the solid obtained by rotating $A$ about the $y$-axis.

(b) ________________

(14 points) 4. Consider the differential equation

$$\frac{dy}{dt} = -\lambda t \left( \frac{y^2 - k^2}{y} \right), \tag{1}$$

where $\lambda$ is a constant, $t \geq 0$, $k > 0$, but $y$ may be positive or negative. Suppose $y(0) = y_0$.

(2 points) (a) Find all solutions $y$ to (1) satisfying $dy/dt = 0$.

(3 points) (b) Find the general solution to (1).

(2 points) (c) Suppose $y_0 = 3k$. Write the solution to (1) in the form $y(t) = \cdots$. What happens as $t \to \infty$?

(2 points) (d) Suppose $y_0 = k/2$. Write the solution to (1) in the form $y(t) = \cdots$. What happens as $t \to \infty$?

(2 points) (e) Suppose $y_0 = -k/2$. Write the solution to (1) in the form $y(t) = \cdots$. What happens as $t \to \infty$?

(3 points) (f) Sketch your solutions, including those from part (a).

(6 points) 5. A professor never finishes his class on time. The time of delay is given by the probability density function

$$p(t) = \frac{k}{1+t^2}, \text{ where } 0 < t < \sqrt{3}$$

(2 points) (a) What is the constant $k$?

(a) ______________

(2 points) (b) What is the average delay?

(b) ______________

(2 points) (c) What is the median delay?

(c) ______________

This is an extra page for answers.