Marks

[24] **1.** Multiple Choice Questions: Select ONE correct answer (a, b, c, d, or e) for each question and write it in the table on the next page. You will not be graded for any work or answers outside those boxes.

(Q1) Which of these integrals does the sum $\displaystyle\sum_{k=1}^{N}\left(\frac{3}{N}\right)\left[\left(-2+\frac{3k}{N}\right)^3-1\right]$ approximate as $N\to\infty$?

(a) $\displaystyle\int_0^3 x^3\,dx$ **(b)** $\displaystyle\int_{-2}^1 (x^3-1)\,dx$ **(c)** $\displaystyle\int_0^3 (x^3-2)\,dx$ **(d)** $\displaystyle\int_{-2}^1 x^3\,dx$ **(e)** $\displaystyle\int_0^1 (x-2)^3\,dx$

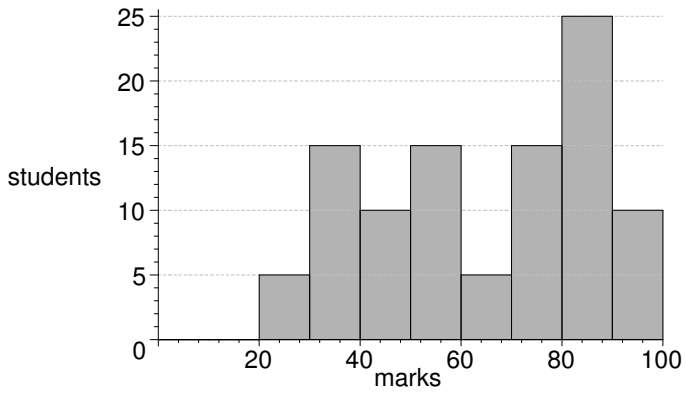
(Q2) You are out fishing. Every time you cast your line, the probability that you catch a fish is 1/4. What is the probability that you catch at least one fish within the first 2 attempts?

(a) 1/16 **(b)** 7/16 **(c)** 1/2 **(d)** 9/16 **(e)** 15/16

(Q3) For which of these differential equations is $y=3$ a steady state solution?

(a) $\dfrac{dy}{dt}=2y-6$ **(b)** $\dfrac{dy}{dt}=y^2+9$ **(c)** $\dfrac{dy}{dt}=\dfrac{1}{3-y}$ **(d)** $\dfrac{dy}{dt}=\cos(\pi y)$ **(e)** $\dfrac{dy}{dt}=3$

(Q4) A Math 103 class of 100 students took a test. The following graph shows how many students' marks were in each of the intervals 1–10, 11–20, ..., 91–100.



What was (approximately) the median?

(a) 50 **(b)** 60 **(c)** 70 **(d)** 80 **(e)** 90

(Q5) A charged molecule moves in a changing electric field which results in an acceleration of $a(t)=\cos(t)$ (in appropriate units). At time $t=0$, the molecule is at rest at position $x(0)=1$. What is the position $x(t)$ when the molecule's velocity reaches the value -1 for the first time after time 0?

(a) -1 **(b)** 0 **(c)** 1 **(d)** 2 **(e)** The velocity never reaches -1

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(Q6) Each mouse born with a certain genetic mutation has probability 0.7 of contracting a certain disease. Out of 10 mice born with the mutation, the probability that exactly 8 will contract the disease is

(a) $45(.7)^8(.3)^2$ **(b)** $0.7(0.3)^7$ **(c)** $1/11$ **(d)** $8/10$ **(e)** $45/2^{10}$

Q1	Q2	Q3	Q4	Q5	Q6

NOTE: carefully check to ensure that you have correctly matched the response with the relevant questions. Only answers in this table using letters a, b, c, d, or e will be graded for Problem 1. Illegible or ambiguous responses will not receive marks.

[6] **2.** Using the fact that

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

for x near zero, find the Taylor series for the function $f(x) = \dfrac{1}{(1-x)^2}$ valid for x near zero.

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[10] **3.** Consider the differential equation

$$\frac{dx}{dt} = \frac{x^2 - 2}{2x}$$

(a) Solve the differential equation by separation of variables.

$x(t) =$ [ ]

(b) Find the solution with the initial condition $x(0) = 2$.

$x(t) =$ [ ]

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[15] **4.** Find the following integrals:

(a) $\displaystyle\int_{-1}^{0} \frac{1}{\sqrt{1-x}}\,dx$

(b) $\displaystyle\int x(\ln(x))^2\,dx$

(c) $\displaystyle\int \frac{x+7}{(x-3)(x+2)}\,dx$

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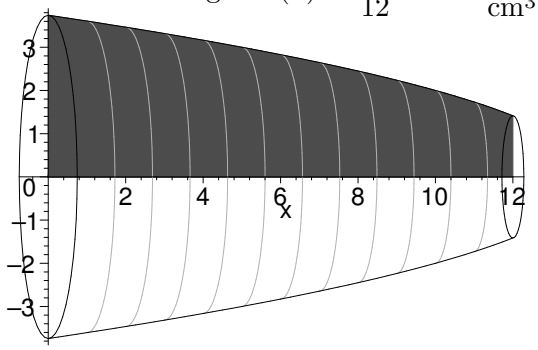
[12] **5.** A carrot has the shape formed by revolving the region under the graph of $f(x) = \sqrt{14 - x}$ for $0 \le x \le 12$ cm about the x axis. The concentration of vitamin A is found to vary in the carrot with x according to $c(x) = \frac{1}{12} e^{-x/12} \ \frac{\text{mg}}{\text{cm}^3}$.



(a) What is the volume of the carrot?

Volume = _____ cm^3

(b) What is the total amount of vitamin A in the carrot?

Vitamin A = _____ mg

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(c) You want to cut the carrot at a certain value of x, obtaining two pieces of equal volume. How many centimetres from $x = 0$ do you have to cut?

Cut at $x =$ _____ cm

[9] **6.** Seeds are dispersed from a tree in all directions uniformly. The density of seeds that land on the ground at a distance r away is approximately described by the function:

$$s(r) = 50 \sin\left(\frac{\pi r}{10}\right) \text{ seeds/m}^2 \quad 0 \leq r \leq 10 \text{ m}$$

(a) At what distance from the tree are the seeds most densely distributed?

Distance = _____ m

(b) What is the total number of seeds on the ground within a 10 m radius of the tree?

There are _____ seeds

Continued on page 8

[12] **7.** *Streptococcus viridans* bacteria divide along a preferred axis and therefore form chains when they divide. You study a population of *S. viridans* in a blood culture. Let X be the length (in μm) of a randomly chosen chain. Its distribution is described by the probability density function

$$p(x) = \frac{1}{x}, \qquad 1 \le x \le \mathrm{e}$$

(a) Find the expected length of a chain (i.e. the expected value of X).

$E[X] =$ μm

(b) What is the probability of finding a chain of length at most 2 μm?

$P\{X \le 2\} =$

(c) Find the standard deviation of X.

$\sigma =$ μm

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[12] **8.** The growth rate of a plant during the first week after germination is given by $g(t) = a\,e^{t/2}$ mm/day, where the time t is measured in days, and a is a constant.

(a) Determine the height of the plant $h(t)$ as a function of the time t, assuming that it was zero at time $t = 0$.

$h(t) =$ [ ]

(b) On the first day, the plant grew 1 mm. Determine the constant a.

$a =$ [ ] mm/day

(c) How much did the plant grow on the fifth day (i.e. between $t = 4$ and $t = 5$)? (If you don't know the constant a, use $a = 1$.)

It grew [ ] mm

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Some useful formulae

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\tan^2(x) + 1 = \sec^2(x)$$

The End

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The University of British Columbia

Sessional Examinations - April 2009

Mathematics 103

Integral Calculus with applications to Life Sciences

H. Pfeiffer: section 201 (MWF 10:00) R. Israel: section 202 (MWF 11:00)
P. Borowski: section 203 (MWF 12:00) C. Hauert: section 204 (MWF 1:00)
B. Vanderlei: section 205 (TTh 8:00) R. Das: section 206 (TTh 3:30)

Closed book examination Time: $2\frac{1}{2}$ hours

Name _____ **Signature** _____

Student Number _____ **Section** _____

Special Instructions:

No calculators, books, notes, electronic devices or other aids. Unless otherwise indicated, show all your work. Answers not supported by calculations or reasoning may not receive credit. Excessively messy work will not be graded. Write your answers in the boxes where these are provided. The last page contains some helpful formulae, and blank space that can be used for rough work that you don't want the markers to look at (e.g. for question 1).

Rules governing examinations

1. Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.

Speaking or communicating with other candidates.

Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1		24
2		6
3		10
4		15
5		12
6		9
7		12
8		12
Total		100