Math 102- Final examination

University of British Columbia

December 14, 2012, 3:30 pm to 6:00 pm

Name (print):

ID number:

Section number:

This exam is "closed book". Calculators or other electronic aids are not allowed.

A		8
B1-3		15
B4-6		8
C.1		4
C.2		4
C.3		10
C.4		6
C.5		8
Total		63

Rules governing examinations

• Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

• Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.

• Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

• Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
(a) speaking or communicating with other candidates, unless otherwise authorized;
(b) purposely exposing written papers to the view of other candidates or imaging devices;
(c) purposely viewing the written papers of other candidates;
(d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
(e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

• Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

• Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

• Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

A. Multiple choice questions

Enter your choice for each multiple choice question **in the box at the bottom of the page**. There are two pages at the end of the exam that can be used for rough work. No partial marks will be given for this section.

A.1 [2 pts] The functions $f(x) = x^2$ and $g(x) = x^3$ are equal at $x = 0$ and at $x = 1$. Between $x = 0$ and $x = 1$, for what value of x are their graphs furthest apart?

(a) $x = 1/2$, (b) $x = 1/3$, (c) $x = 2/3$, (d) $x = 1/4$, (e) $x = 3/4$.

A.2 [2 pts] To find a point $(a, \cos(a))$ on the graph of $y = \cos(x)$ whose tangent line goes through the origin, which of the following equations must you solve?

(a) $\cot(a) = a$, (b) $\cot(a) = -a$, (c) $\tan(a) = a$, (d) $\tan(a) = -a$.

A.3 [2 pts] Which of the following statements concerning the limit below is necessarily true?

$$L = \lim_{x \to -\infty} \frac{x^n - 2x^3 + 2}{x^3 + 4}$$

(a) If $n < 3$ then $L = 0$.
(b) If $n = 3$ then $L = -2$.
(c) If $n > 3$ and n is odd then $L = -\infty$.
(d) If $n > 3$ and n is even then $L = -\infty$.

A.4 [2 pts] The transmembrane potential in a neuron is well-described by the equation

$$\frac{dv}{dt} = -v^3 + 20v^2 + 3500v.$$

If the transmembrane potential starts at $v(0) = 10$, what value (v_∞) does it approach as $t \to \infty$?

(a) $v_\infty = -70$, (b) $v_\infty = -50$, (c) $v_\infty = 0$, (d) $v_\infty = 50$, (e) $v_\infty = 70$.

Answers:

Q A.1	Q A.2	Q A.3	Q A.4

B. Short answer and fill-in-the-box questions

A correct answer in a box will get full points. Partial marks may be given if you show your work.

B.1 [4 pts] For the following questions, refer to the slope fields in the figure below. Match the plots below with the differential equations by filling in the box with A, B, C, D, or NOT (for "None Of These").

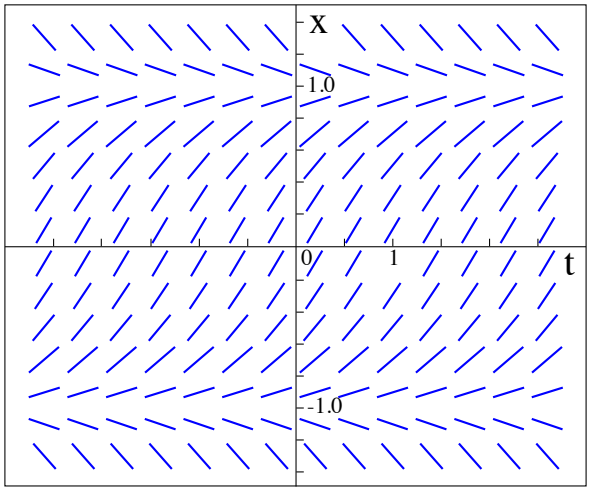
(a) The equation $\frac{dx}{dt} = 1 - t^2$ matches the slope field $\boxed{}$.

(b) The equation $\frac{dx}{dt} = t(t-1)(t+1)$ matches the slope field $\boxed{}$.

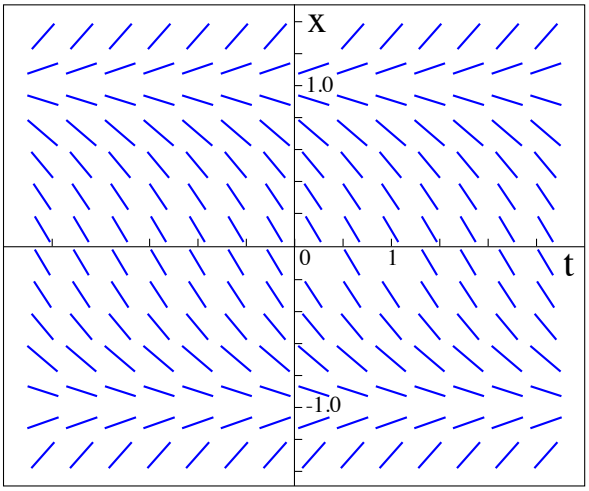
(c) The equation $\frac{dx}{dt} = x^2 - 1$ matches the slope field $\boxed{}$.

(d) The equation $\frac{dx}{dt} = 1 - x^2$ matches the slope field $\boxed{}$.

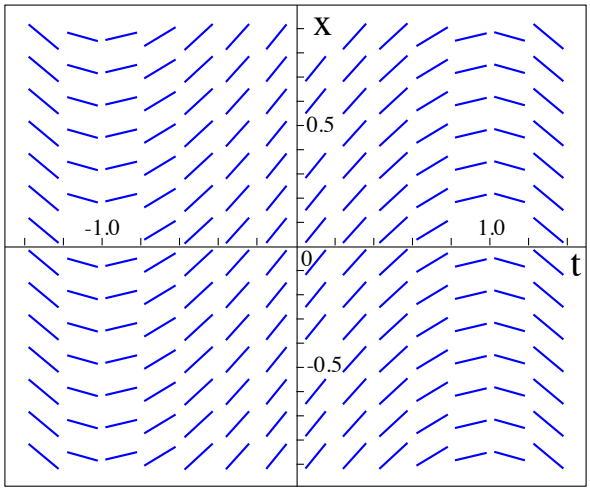
A



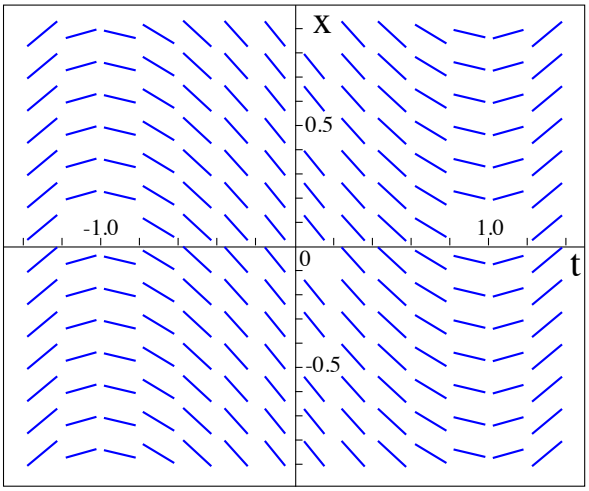
B



C



D



B.2 [8 pts] Consider the function

$$f(x) = \frac{x}{x^2+1}$$

defined on the whole real line.

The zeros of f', in increasing order, are $c_1 =$ ☐ and $c_2 =$ ☐.

The zeros of f'', in increasing order, are $r_1 =$ ☐, $r_2 =$ ☐ and $r_3 =$ ☐.

In each empty cell of the tables below, enter a $+$ or $-$ to indicate the sign of f, f' and f'' as appropriate.

	$(-\infty, 0)$	0	$(0, \infty)$
$f(x)$		0	

	$(-\infty, c_1)$	c_1	(c_1, c_2)	c_2	(c_2, ∞)
$f'(x)$		0		0	

	$(-\infty, r_1)$	r_1	(r_1, r_2)	r_2	(r_2, r_3)	r_3	(r_3, ∞)
$f''(x)$		0		0		0	

B.3 [3 pts] A researcher measures the length of 16 adult wombats and finds a sample mean of 57 cm and a sample standard deviation of 12 cm. The standard error of the mean is estimated to be 3 cm. If she plans to carry out measurements on 100 more wombats, the following can be expected (place an X in exactly one column for each row):

	decrease.	stay roughly the same.	increase.
The sample mean will			
The sample maximum will			
The standard error of the mean will			

B.4 [2 pts] A car sales person works on commission and gets paid \$2,000 for every car sold. In any month, there is a 0.7 probability of selling exactly one car, a 0.2 probability of selling exactly two cars and a 0.1 probability of selling exactly three cars. Nobody has ever sold more than 3 cars in a month. He gets a bonus of \$1000 if he sells three cars in a single month. What is the sales person's expected monthly income?

Expected income = $\boxed{}$

B.5 [3 pts] In the Chernobyl reactor explosion, which occurred on April 26, 1986, substantial amounts of the isotope strontium-90 (^{90}Sr) contaminated the area around the nuclear plant. ^{90}Sr decays at a rate proportional to its quantity. ^{90}Sr has a half-life of 29 years; that is, it takes 29 years for a quantity of ^{90}Sr to decrease by half. What is the proportion of ^{90}Sr originally released which remain on April 26, 2012?

Proportion of original amount remaining on April 26, 2012 = $\boxed{}$

B.6 [3 pts] There are six neurons connected to a muscle. If four or more of the neurons fire at the same time, the muscle contracts. In any millisecond, each neuron has a 0.3 probability of firing. What is the probability that the muscle contracts in any particular millisecond? You don't have to simplify your answer.

Prob(muscle contracts) = $\boxed{}$

C. Long Answer Problems

C.1 [4 pts] Suppose $f(x)$ satisfies the equation $\ln(f(x)) = x \ln x$. Express $f'(x)$ in terms of x only (i.e. do not leave $f(x)$ in the expression).

C.2 [4 pts] Use the definition of the derivative to calculate the derivative of $g(x) = \sqrt{x}$. The following might be a useful fact in simplifying the limit: $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$.

C.3 [10 pts] This question is about approximating the value of $b = \sqrt{48}$.

(a) Between which two successive integers does b lie?

$$\square < b < \square$$

(b) Use a linear approximation to estimate b. Is your estimate larger or smaller than the actual value? Justify your claim.

(c) Determine an appropriate function that has a zero at b and apply one iteration of Newton's method (i.e. find x_1) to estimate b starting with an initial guess x_0 that is the nearest integer to the actual value.

C.4 [6 pts] Dead leaves accumulate on the ground in a forest at a rate of 5 grams per square centimeter per year. At the same time, these leaves decompose at a continuous rate of 50 percent per year.

(a) Write a differential equation for the total mass $Q(t)$ of dead leaves (per square centimeter) at time t.

$$\frac{dQ}{dt} = \boxed{}.$$

(b) What is the steady state mass of leaves per square centimeter?

$Q_{ss} =$ $\boxed{}$ grams per square cm.

(c) Sketch the graph of the solution to your differential equation showing that the mass of dead leaves tends toward a steady state assuming that initially ($t = 0$) there are no leaves on the ground.

Choose ONE of the next two problems. Only one of them will be marked. Indicate which one should be marked by placing an X in the box next to the question to be marked.

C.5.1 [8 pts] A hot air balloon with a basket hanging below it is released from the ground and rises straight up at a speed of 5 meters per second. At the moment the balloon is released, a girl is 29 meters from the point on the ground directly below the balloon and is riding her bicycle toward the ballon at a speed of 2 meters per second. From the perspective of the girl, at what time t is the angle between the ground and the basket hanging below the balloon changing most quickly?

$t =$

C.5.2 [8 pts] An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?

Minimizing ratio:

This page and the following one are for rough work. If you fill up the space available for a B or C question, you can continue here but state clearly on the page where the question is asked that there is work here to be marked. This page will not be looked at unless you indicate that it should be.

This page is for rough work.