Marks

[8] **1.** Using the definition of derivative, find $f'(2)$ where $f(x) = \dfrac{1}{1+x^2}$.

Continued on page 3

[12] **2.** Use implicit differentiation to find the points on the curve

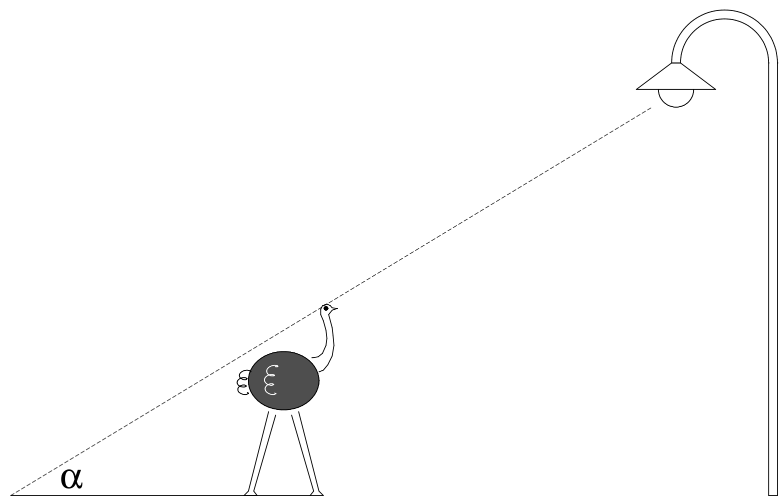
$$3y^3 - 2x^3 - 6x^2y + 5y = 0$$

where the tangent line to the curve is horizontal. Write your answers in the box in the form (x, y).

Points:

Continued on page 4

[14] **3.** An ostrich 1.5 m tall is walking toward a street light 4 m above the ground at a speed of 5 m/s. How fast is the length of the ostrich's shadow decreasing? At what speed is the tip of the shadow moving? If α is the angle between the light and the ground measured at the tip of the shadow, as shown in the picture, what is the rate of change of α when the shadow is 1.5 m long?



Length of shadow decreasing at:

Speed of tip of shadow:

Rate of change of α:

Continued on page 5

[6] **4.** Suppose the number of bacteria in a colony doubles every 20 minutes. If there are 2^{39} cells (about 5.5×10^{11}) after 12 hours, how many were there at the beginning?

Initial number = $\boxed{}$

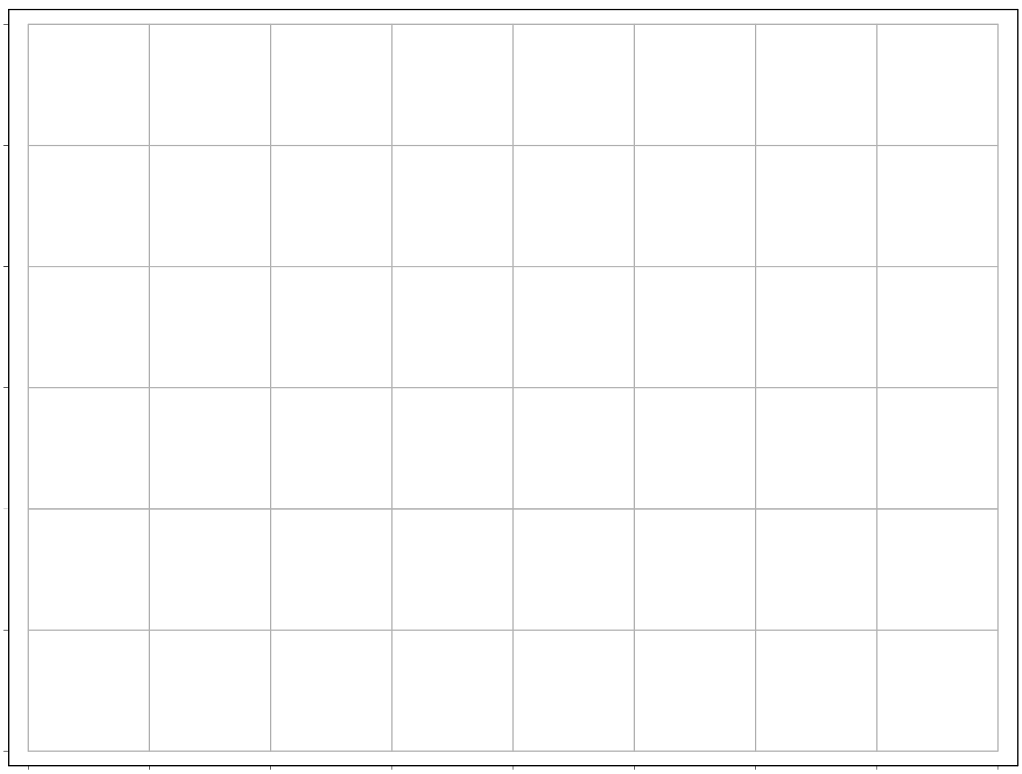
[6] **5.** The tide on a certain shore on the planet Outer Thebulon IV has a period of 36.5 hours, and the high tide level is 8 m above the low tide level. At $t = 0$ the water level is 2 m above the low tide level and rising. Using trigonometric functions, find a function to describe the height $H(t)$ of the water above the low tide level.

$H(t) = \boxed{}$

Continued on page 6

[12] **6.** Sketch the graph of $f(x) = \dfrac{x}{1+x^2}$ on the grid provided, showing all of the following if they are present:

i) x and y intercepts
ii) critical points
iii) intervals where f is increasing or decreasing
iv) points of inflection
v) intervals where f is concave up or down.



Continued on page 7

7. Write your answers in the boxes.

[3] (a) An object moves along a line. If the object's position at time t is $x(t) = 3t^2 - 2t + 1$, what is its average velocity from 0 to time 3? Time is measured in seconds, position in metres.

Average velocity = $\boxed{}$

[3] (b) Use a linear approximation to estimate $(65)^{1/3}$, given that $(64)^{1/3} = 4$. The answer should use fractions, not decimals.

$(65)^{1/3} \approx \boxed{}$

[3] (c) In trying to solve the equation $x^3 + 5x - 3 = 0$ using Newton's method, our initial guess is $x_0 = 1$. What is x_1? The answer should use fractions, not decimals.

$x_1 = \boxed{}$

[3] (d) Using Euler's method to solve the differential equation $y' = y^2$ with $y(0) = 2$ and step size 0.1, what is the approximate value of $y(0.2)$? The answer should be given to at least 3 decimal places.

$y(0.2) \approx \boxed{}$

Continued on page 8

[6] **8.** Find a and b such that $f(x) = ax^3 + bx^2 + 1$ has an inflection point at $(-1, 2)$.

$a = \boxed{}$, $b = \boxed{}$

Continued on page 9

[10] **9.** Okapis have two types of food (**A** and **B**) available in their environment. These animals spend 10 hours every day looking for food. When looking for **A**, the okapi gets 2 kilograms of **A** per hour it spends on this; when looking for **B**, the okapi gets 1 kilogram of **B** per hour. The nutrition value obtained from x kilograms of **A** is given by $x^3 - 16x^2 + 25x + 500$ and from y kilograms of **B** is given by y^2. How should an okapi divide its time between the two types of food to maximize the total daily nutrition value, and what is the total nutrition value when it does so?

Time spent looking for **A**: $\boxed{}$

Total nutrition value: $\boxed{}$

Continued on page 10

10. Consider Newton's law of cooling

$$\frac{dT}{dt} = 2 - \frac{1}{5}T$$

with initial condition $T(0) = 37$.

[6] (a) Find values of the constants a, b and k such that

$$T(t) = a + be^{-kt}$$

is a solution to the initial value problem given above.

[4] (b) Using the solution obtained in (a) to find the time τ at which $T(\tau) = 13$. Express the answer in terms of m, where $m = \ln 3$.

[4] (c) What is the steady state for this differential equation? Is it stable or unstable?

Continued on page 11

Useful Formulae

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Trig identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Values:

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0

The End

Be sure that this examination has 11 pages including this cover

The University of British Columbia
Sessional Examinations - December 2006

Mathematics 102
Differential Calculus with applications to Life Sciences

Closed book examination | Time: $2\frac{1}{2}$ hours

Name ______________________ **Signature** ______________________

Student Number____________________ **Section**______________________

Special Instructions:

Non-graphing calculators allowed, no other aids. Show your work. The last page contains some helpful formulae.

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

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(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		8
2		12
3		14
4		6
5		6
6		12
7		12
8		6
9		10
10		14
Total		100