Final Exam—Math 101, Section 101

December 6, 2014

Duration: 150 minutes

Name: _____ Student Number: _____

Do not open this test until instructed to do so! This exam should have 12 pages, including this cover sheet. No textbooks, notes, calculators, or other aids are allowed.

UBC rules governing examinations:

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (a) speaking or communicating with other examination candidates, unless otherwise authorized;
   (b) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (c) purposely viewing the written papers of other examination candidates;
   (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Problem	**Out of**	**Score**
1	7	
2	18	
3	19	
4	21	
5	29	
6	8	
7	12	
Total	114	

1. **True/False**. No justification is required; just choose the correct answer. Each correct answer is worth one point.

(a) **[1 pt]** The limit of the sequence $\{\frac{(-1)^n(n^4-2n^2+3)}{8n^4+6n^3-5}\}$ is $\frac{1}{8}$.

True ☐ **False** ☐

(b) **[1 pt]** If $f'(x)$ is continuous on $[1,\pi]$ then $\int_1^\pi f'(x)dx = f(\pi) - f(1)$.

True ☐ **False** ☐

(c) **[1 pt]** $\int_{-1}^{1} \frac{1}{x^2}dx = 0$.

True ☐ **False** ☐

(d) **[1 pt]** If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges.

True ☐ **False** ☐

(e) **[1 pt]** If f is continuous on $[a,b]$ then $\int_a^b xf(x)dx = x\int_a^b f(x)dx$.

True ☐ **False** ☐

(f) **[1 pt]** $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$.

True ☐ **False** ☐

(g) **[1 pt]** $\int_1^\infty \frac{1}{x^\pi}dx$ is convergent.

True ☐ **False** ☐

2. **Short Answer**. Each question is worth 3 points, but not all questions are the same difficulty. At most one point will be given for incorrect answers. Show all work and write your answers in the box provided.

(a) **[3 pts]** Evaluate $\int \frac{x}{x^2-3} dx$.

Answer

(b) **[3 pts]** Differentiate $\int_x^{e^x} \sqrt{\sin t} dt$.

Answer

(c) **[3 pts]** Let $R_n = \sum_{i=1}^{n} \frac{ie^{i/n}}{n^2}$. Express $\lim_{n\to\infty} R_n$ as a definite integral. *Do not evaluate this integral.*

Answer

(d) **[3 pts]** Evaluate $\int_0^{\pi} \cos^2 x dx$.

Answer

(e) **[3 pts]** Find the sum of the series $\sum_{n=0}^{\infty} \frac{1+3^{n+1}}{4^n}$.

Answer

(f) **[3 pts]** Give the first two nonzero terms in the Maclaurin series for $\int \frac{e^{-x^2}-1}{x} dx$.

Answer

Full Solution Problems. In questions 3-7, show all of your work and write your final answers in the box provided.

3.

(a) **[7 pts]** Determine, with explanation, whether the series $\sum_{n=1}^{\infty} \frac{5^k}{4^k+3^k}$ converges or diverges.

Answer

(b) **[12 pts]** Find the *interval of convergence* for the series $\sum_{n=1}^{\infty}(-1)^n n^2(x-a)^{2n}$ where a is a constant.

Answer

4.

(a) **[11 pts]** Find the y-coordinate of the centroid of the region bounded by $y = e^x$, $x = 0$, $x = 1$, and $y = -1$.

Answer

(b) **[10 pts]** Calculate the volume of the solid generated by rotating the region from part (a) about the line $y = -1$.

Answer

5.

(a) **[12 pts]** Using a limit of right-endpoint Riemann sums, evaluate $\int_1^4 (2x-1)dx$. No credit will be given for the use of anti-differentiation, but you may use it to check your answer. You may use the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Answer

(b) **[7 pts]** Evaluate $\int \frac{5x+1}{x^2+5x+6}dx$.

Answer

(c) **[10 pts]** Evaluate $\int \frac{1}{x^2\sqrt{x^2+16}}dx$.

Answer

6. **[8 pts]** A variable force $F(x) = \frac{a}{\sqrt{x}}$ Newtons moves an object along a straight line when it is a distance of x meters from the origin. If the work done in moving the object from $x = 1$ meters to $x = 16$ meters is 18 Joules, what is the value of a?

Answer

7. **[12 pts]** A tank contains 1000 litres of pure water. A solution that contains 0.01 kg/L of salt is poured into the tank at a rate of 40 L/min. The tank is kept thorughly mixed and the contents drain out at the same rate. How much salt is in the tank after 2 hours?

Answer