The University of British Columbia

Final Examination - April 24, 2009

Mathematics 101

All Sections

Closed book examination Time: 2.5 hours

Last Name _____ **First** _____ **Signature** _____

Student Number _____

Section : _____

Instructor : _____

Special Instructions:

No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form, where calculator means basic scientific calculator. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBC-card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		30
2		6
3		10
4		12
5		10
6		12
7		10
8		10
Total		100

1. **Short-Answer Questions.** Put your answers in the boxes provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. At most one mark will be given for an incorrect answer. **Unless otherwise stated, simplify your answers as much as possible.**

Marks

[3] (a) Evaluate $\int \frac{3+x^5}{\sqrt{x}}dx$

Answer

(b) What integral is defined by the following expression? $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{4n}\tan\frac{i\pi}{4n}$

[3] *Do not evaluate the integral.*

Answer

[3] (c) Evaluate $\int_0^1 (y+1)\sqrt{2y+y^2}dy$

Answer

[3] (d) Evaluate $\int x^2 \ln x \; dx$

Answer

[3] (e) Find the length of the curve $x = 100 + 2y^{3/2}$, $0 \leq y \leq 11$.

Answer

[3] (f) What is the average value of $|\sin\theta - \cos\theta|$ over the interval $0 \leq \theta \leq \pi/2$?

Answer

[3] (g) Give the first three nonzero terms of the Maclaurin series (power series in x) for

$$\int \frac{e^{-x^2}-1}{x}dx$$

Answer

[3] (h) For what values of r does the function $y = x^r$ satisfy the following differential equation (for $x > 0$)?

$$x^2y'' + 4xy' + 2y = 0$$

Answer

[3] (i) Calculate the derivative $f'(x)$ if $f(x) = \int_x^{e^x} \sqrt{\cos t}\, dt$.

Answer

(j) Give the Simpson's rule approximation to $\int_0^2 \sin(e^x)\, dx$ using 4 equal subintervals (do
[3] not simplify your answer).

Answer

Full-Solution Problems. In the remaining questions, justify your answers and **show all your work.** If a box is provided, write your final answer there. If you need more space, use the back of the previous page. **Unless otherwise indicated, simplification of answers is not required.**

[6] 2. For what values of p does $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^p}$ converge?

Answer

[10] 3. Let R be the finite region in the xy-plane bounded by $x = 0$, $y = 0$ and $y = \cos x$.

(a) Calculate the centroid of R.

Answer

(b) Express the volume of the solid obtained by rotating R about the line $x = -2$ as an integral. *Do not evaluate the integral.*

Answer

4. Calculate the integrals:

[6] (a) $\int (4 - x^2)^{-3/2}\, dx$

Answer

[6] (b) $\int \sqrt{1 + e^x}\, dx$

Answer

[10] 5. Let X be a random variable with probability density function

$$f(x) = \begin{cases} kx(1-x^4) & \text{if } 0 \le x \le 1, \\ 0 & \text{if } x < 0 \text{ or } x > 1. \end{cases}$$

(a) Find the value of k.

Answer

(b) Find the mean μ.

Answer

(c) Find an algebraic equation satisfied by the median m.

Answer

6. Solve these differential equations:

[6] (a) $y' = xy^2$ with initial conditions $y(0) = 1$.

Answer

[6] (b) $y' - 2y = 4 + e^{3t}$. (*Give the general solution*)

Answer

[10] 7. An open metal tank has two ends which are isosceles triangles with vertex at the bottom, two sides which are rectangular, and an open top. The tank is 1 metre wide, 2 metres deep. 10 metres long and full of water ($density = 1000kg/m^3$)

(a) What is the hydrostatic force on each triangular end?

Answer

(b) Express the work required to pump the water out of the tank, if it is pumped out over the top edge, as an integral. (*Do not evaluate the integral.*)

Answer

(c) Express the hydrostatic force on each of the rectangular sides as an integral. (*Do not evaluate the integral.*)

Answer

[10] 8. Consider the chemical reaction

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C} + \mathrm{D}.$$

Suppose at time $t = 0$ sec the concentration of chemical A is 0.1 mol/L, the concentration of chemical B is 0.2 mol/L, and the concentrations of chemicals C and D are both 0. For $t \geq 0$, let $x(t)$ be the concentration of chemical D in mol/L. It can be shown that $x(t)$ is the solution to the initial-value problem

$$\frac{dx}{dt} = k(0.1 - x)(0.2 - x), \qquad x(0) = 0,$$

where k is a positive constant whose value can be determined by experiment.

(a) Solve the initial-value problem to find $x(t)$ explicitly.

Answer

(b) What value does the concentration of chemical D approach as t approaches ∞?

Answer