Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - April 2007

Mathematics 101

Integral Calculus with Applications to Physical Sciences and Engineering

Closed book examination Time: 2.5 hours

Last Name: _____ **First Name:** _____

Student Number: _____ **Instructor's Name:** _____

Signature: _____ **Section Number:** _____

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.
2. Read and observe the following rules:
No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
(b) Speaking or communicating with other candidates.
(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
3. Smoking is not permitted during examinations.

1		33
2		20
3		20
4		12
5		8
6		7
Total		100

Marks

[33] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, simplify your answer as much as possible.

(a) Evaluate $\int (2y+1)^5 \, dy$.

Answer

(b) Evaluate $\int_{-1}^{0} (2x - e^x) \, dx$.

Answer

(c) Express $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$ as a definite integral. *Do not* evaluate this integral.

Answer

Continued on page 3

(d) Write down the Simpson's Rule approximation S_4 for $\displaystyle\int_0^4 \frac{1}{1+x^3}\,dx$. You may leave your answer expressed as a sum of fractions.

Answer

(e) Calculate the volume of the solid obtained by rotating the region above the x-axis, below the curve $y = (\sin x)/x$, and between the lines $x = \pi/2$ and $x = \pi$ about the y-axis.

Answer

(f) Write the *form* of the partial-fraction decomposition for

$$\frac{10}{(x+1)^2(x^2+9)}$$

Do not determine the numerical values of the coefficients.

Answer

(g) An exponentially distributed continuous random variable X has probability density function $f(x) = ke^{-kx}$, for $x \geq 0$, where k is a positive constant. The *median* value of X equals 10. Find k.

Answer

Continued on page 4

(h) Find the length of the curve $y = 1 + (2/3)x^{3/2}$, $0 \le x \le 1$.

Answer

(i) Evaluate $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\, dx$.

Answer

(j) Find the first three nonzero terms in the power series representation in powers of x (i.e. the Maclaurin series) for $\displaystyle\int_0^x t\cos(t^3)\, dt$.

Answer

(k) Let $f(x) = \displaystyle\int_{e^x}^0 \cos^3 t\, dt$. Find $f'(x)$.

Answer

Continued on page 5

Full-Solution Problems. In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Unless otherwise indicated, simplification of answers is not required.

[20] **2.** (a) [5] Sketch the bounded region that lies between the curves $y = 4 - x^2$ and $y = (x-2)^2$, and find its area. (Place only your answer for the area in the answer box.)

Answer

(b) [5] Find the numbers b such that the average value of the function $f(x) = 3x^2 - 6x + 2$ on the interval $[0, b]$ is equal to 0.

Answer

Continued on page 6

(c) [4] Set up, but do not evaluate, a definite integral for the volume of the solid obtained by rotating the region between $y = 0$ and $y = \sin x$, for $0 \leq x \leq \pi$, about the line $y = 1$.

Answer

(d) [6] Let R be the region under the curve $y = e^{-x}$ and above the x-axis, for $0 \leq x \leq 1$. Find the x-coordinate of the centroid (centre of mass) of R.

Answer

Continued on page 7

[20] **3.** Evaluate the following integrals.

(a) [4]

$$\int \frac{x}{\sqrt{1-x^4}}\,dx$$

Answer

(b) [6]

$$\int_0^1 \frac{2x+3}{(x+1)^2}\,dx$$

Answer

Continued on page 8

(c) [6]

$$\int \frac{dx}{\sqrt{x^2+2x+5}}$$

Answer

(d) [4]

$$\int (\cos^3 x)(\sin^4 x)\, dx$$

Answer

Continued on page 9

[12] **4.** (a) [4] Find the general solution of the differential equation $4y'' + y' = 0$.

Answer

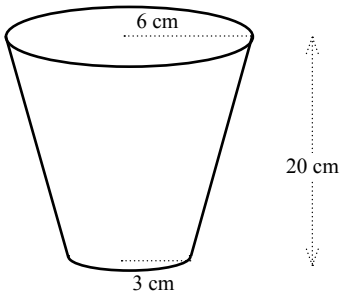
(b) [8] Solve the initial-value problem $y'' - 4y' + 5y = 5x^2 - 3x - 2,\ y(0) = 0,\ y'(0) = 2$.

Answer

Continued on page 10

[8] **5.** A paper cup has the shape depicted below. All of its horizontal cross sections are circles; the radius of the cup's bottom is 3 cm and the radius of its top is 6 cm. The cup is full of Cona Cola, which has a density of 1000 kg/m^3. The Cona Cola is drunk through a vertical straw that extends 10 cm above the top of the cup and reaches the bottom of the cup. Express as an explicit definite integral the work performed in drinking all the cola. Do *not* evaluate this integral. For the acceleration due to gravity use the value 9.8 m/sec^2.

Answer



Continued on page 11

[7] **6.** The population of fish in a lake is m million, where $m = m(t)$ varies with time t (in years). The number of fish is currently 2 million.

(a) [3] Suppose m satisfies the logistic-growth differential equation

$$\frac{dm}{dt} = 16m\left(1 - \frac{m}{4}\right)$$

When will the number of fish equal 3 million? You may use the fact that the general solution to the logistic-growth differential equation $y' = ky(1 - (y/K))$ is $y = K/(1 + Ae^{-kt})$, where A is a constant.

Answer

(b) [4] Suppose instead that (because of fishing by humans) m satisfies

$$\frac{dm}{dt} = 16m\left(1 - \frac{m}{4}\right) - 12$$

Will the fish population ever equal 3 million? You must give justification for your answer.

Answer

The End