

Be sure that this examination has 12 pages including this cover

The University of British Columbia  
Sessional Examinations - December 2007

Mathematics 100/180

*Differential Calculus with Applications to Physical Sciences and Engineering*

Closed book examination

Time: 2.5 hours

Last Name: \_\_\_\_\_ First Name: \_\_\_\_\_

Student Number: \_\_\_\_\_ Instructor's Name: \_\_\_\_\_

Signature: \_\_\_\_\_ Section Number: \_\_\_\_\_

**Rules governing examinations**

1. Each candidate should be prepared to produce his or her library/AMS card upon request.
2. Read and observe the following rules:  
No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.  
Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.  
CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - (a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
  - (b) Speaking or communicating with other candidates.
  - (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
3. Smoking is not permitted during examinations.

1		42
2		10
3		10
4		16
5		12
6		6
7		4
Total		100

Marks

- [42] 1. **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

Answer

(b) Evaluate  $\lim_{x \rightarrow -\infty} \frac{1 - x - x^2}{2x^2 - 7}$

Answer

(c) Find the derivative of  $\frac{1}{x + ae^x}$ , where  $a$  is a constant.

Answer

- (d) Find the equation of the tangent line to the curve  $y = 1 + x + \sin x$  at the point  $(0, 1)$ . Write your answer in the form  $y = mx + b$ , where  $m$  and  $b$  are integers.

Answer

- (e) Find the derivative of  $e^{\cos(x^2)}$ .

Answer

- (f) For the curve defined by the equation  $\sqrt{xy} = x^2y - 2$ , find the slope of the tangent line at the point  $(1, 4)$ .

Answer

- (g) If  $f(x) = (\cos x)^{\sin x}$ , find  $f'(x)$ .

Answer

- (h) Use a linear approximation to estimate  $(2.001)^3$ . Write your answer in the form  $n/1000$ , where  $n$  is an integer.

Answer

- (i) Compute  $\lim_{x \rightarrow 0} \frac{x + \cos(2x) - e^x}{x^2}$ . *Hint:* Use Maclaurin series (you may, however, use any method you know).

Answer

- (j) If  $f(x) = \sin(x^2)$ , compute  $f^{(6)}(0)$ . *Hint:* Use Maclaurin series.

Answer

- (k) Find the absolute maximum *value* of  $f(x) = x/(x^2 + 1)$  on the interval  $[0, 2]$ .

Answer

- (l) The function  $f(x) = b/(x^2 + ax + 2)$  has a local maximum at  $x = 1$ , and the local maximum value  $f(1)$  equals 2. Find the values of  $a$  and  $b$ .

Answer

- (m) Newton's Method is used to approximate a positive solution of the equation  $\sin x = x^2$ , starting with the initial approximation  $x_1 = 1$ . Find  $x_2$ . *Remember:* You do not need to simplify your answers.

Answer

- (n) Given that  $f''(x) = 24x^2 + 6x + 10$ ,  $f(1) = 10$ , and  $f'(1) = 20$ , find  $f(x)$ .

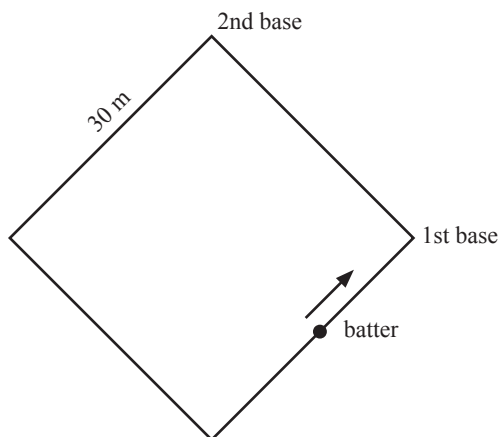
Answer

**Full-Solution Problems.** In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required unless explicitly requested.

- [10] **2.** When a cold drink is taken from a refrigerator, its temperature is  $5^{\circ}\text{C}$ . After 25 minutes in a room that has fixed temperature  $20^{\circ}\text{C}$ , its temperature has increased to  $10^{\circ}\text{C}$ . What is the temperature of the drink after another 25 minutes, i.e. 50 minutes after the drink is taken from the refrigerator? Assume that the temperature of the drink satisfies Newton's Law of "Cooling." To receive full marks, write your answer in completely simplified form, with **no** "ln"s appearing.

Answer
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- [10] **3.** A baseball diamond is a square with side length 30 m, as shown in the diagram below. A batter hits the ball and runs toward 1st base with a speed of 8 m/s. When the batter is halfway to 1st base, at what rate is his distance from 2nd base changing? *Remember:* You do not need to simplify your answers.



Answer

[16] 4. Let  $f(x) = x\sqrt{3-x}$ .

(a) (2 marks) Find the domain of  $f(x)$ .

Answer
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(b) (4 marks) Determine the  $x$ -coordinates of the local maxima and minima (if any) and intervals where  $f(x)$  is increasing or decreasing.

(c) (2 marks) Determine intervals where  $f(x)$  is concave upwards or downwards, and the  $x$ -coordinates of inflection points (if any). You may use, without verifying it, the formula  $f''(x) = (3x - 12)(3 - x)^{-3/2}/4$ .

Question 4 continued on the next page...

Continued on page 9



## Question 4 continued

- (d) (2 marks) There is a point at which the tangent line to the curve  $y = f(x)$  is vertical. Find this point.

Answer
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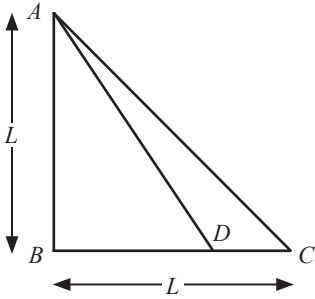
- (e) (2 marks) The graph of  $y = f(x)$  has no asymptotes. However, there is a real number  $a$  for which  $\lim_{x \rightarrow -\infty} \frac{f(x)}{|x|^a} = -1$ . Find the value of  $a$ .

Answer
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- (f) (4 marks) Sketch the graph of  $y = f(x)$ , showing the features given in items (a) to (d) above and giving the  $(x, y)$  coordinates for all points occurring above and also all  $x$ -intercepts.

- [12] 5.  $ABC$  is a right triangle, with right angle at the point  $B$  and sides  $AB$  and  $BC$  both having fixed length  $L$ . A man wants to go from point  $C$  to point  $A$  by first walking to some point  $D$  between  $B$  and  $C$  and then walking directly to the point  $A$ . He can walk with velocity  $v_1$  from  $C$  to  $D$  and velocity  $v_2$  from  $D$  to  $A$ . If  $v_1 = 2v_2$ , find the distance of  $D$  from  $B$  that minimizes the time the man spends getting from  $C$  to  $A$ .

Answer



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- [6] **6.** Find an equation of a line that is tangent to both of the curves  $y = x^2$  and  $y = x^2 - 2x + 2$  (at different points).

Answer
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[4] **7.** Let  $f(x) = x|x|$ .

(a) (2 marks) *Using the definition of the derivative*, show that  $f(x)$  is differentiable at  $x = 0$ .

(b) (2 marks) Find the second derivative of  $f(x)$ . Explicitly state, with justification, the point(s) at which  $f''(x)$  does not exist, if any.