The University of British Columbia
Department of Mathematics
Qualifying Examination—Analysis
January 6, 2018

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1. (10 points) (a) Determine if the following series converges or diverges:
$$\sum (-1)^n \frac{\sqrt{n}}{1+\sqrt{n}}$$
(b) For which values of $\alpha > 0$ does the following series converge?
$$\sum \frac{1}{n^\alpha (\log_2 n)^2}$$
(c) Find the radius of convergence of the following power series:
$$\sum \frac{z^n}{(1+(-1)^n)2^n + (1-(-1)^n)3^n}$$

2. (10 points) Let S be the part of the cylinder $(x+y+1)^2 + z^2 = 4$ which lies in the first octant. Find the flux of $\vec{F}$ upwards through S where
$$\vec{F} = xy\,\hat{\boldsymbol{\imath}} + (z - xy)\,\hat{\boldsymbol{\jmath}}.$$

3. (10 points) Let I be a bounded interval in $\mathbb{R}$ and f_n be continuous functions on I such that $f_{n+1}(x) \leq f_n(x)$ for all $x \in I$, $n \in \mathbb{N}$. Suppose that $f_n(x)$ converges to 0 for each $x \in I$.
(a) Give a counterexample to show that the conditions above do not imply that $f \to 0$ uniformly.
(b) Suppose that I is compact. Prove that $f \to 0$ uniformly.

4. (10 points) How many zeros does the polynomial $z^4 + \frac{1}{4}z^3 - \frac{1}{4}$ have in the annulus $\{z \in \mathbb{C}\colon \frac{1}{2} < |z| < 1\}$?

5. (10 points) Determine for which integer values of n (positive, negative, or 0), there exists a holomorphic function defined in the region $|z| > 1$, whose derivative is
$$\frac{z^n}{1+z^2}\,.$$

6. (10 points) Let $D = \{z \in \mathbb{C}\colon |z| < 1\}$, and suppose that $f : D \to \mathbb{C}$ is holomorphic, and injective when restricted to $D \setminus \{0\}$. Prove that f is injective.