Math 605D Tensor Decompositions and Applications

Fall 2020 - Syllabus

With the emergence of big data, it is more and more often the case that we encounter tensor-shaped data. The importance of being able to decompose a tensor is (at least) two-fold. First, finding the decomposition provides hidden information about the data at hand, and second, having a concise decomposition of the tensor allows us to store it much more efficiently. One of the biggest obstacles in dealing with tensors, however, is that decomposing them is often computationally hard.

This research-oriented course will introduce tensors (or multi-dimensional arrays) and their uses in statistics, machine learning, and the sciences. In particular, we will illustrate fundamental theoretical properties of several types of tensor decompositions, including CP-decomposition, nonnegative matrix and tensor decomposition, Tucker decomposition as well as tensor network decompositions arising from physics. We will see how these naturally come up in hidden variable models, Gaussian mixture models, directed and undirected graphical models, blind source separation, independent component analysis, and quantum physics. We will discuss algorithms for computing such decompositions, and will exhibit open problems.

Instructor: Elina Robeva, `erobeva@math.ubc.ca`, URL: `https://math.ubc.ca/~erobeva/`.

Class time: TTh 9:30 - 10:50 Pacific Time.

Class location: Zoom

Class website: `https://sites.google.com/view/ubc-math-605d/`

Prerequisites: Besides general mathematical maturity, the minimal suggested requirements for the course are linear algebra (e.g., one of Math 221, 223, 307), and basic probability (e.g., one of Math 302, 318). Some familiarity with machine learning is encouraged, but not required.

Bibliography: We will use a variety of book chapters and current papers. Some of these are listed at the end of this syllabus.

Lecture notes: Lecture notes and homework will be posted on the course website.

Grades: Research project 50%, Homework 40%, Scribing 5%, Participation 5%.

Research Project: This course includes a research project in which students address a topic of their choice. A one-page abstract describing the goals of the project, the main questions, and the approaches the students plan to take, is due midway through the term. Final presentations are during the last two lectures of the course and a final write-up of the project of maximally 10 pages is due at the end of the term. Students, preferably of different backgrounds, can pair up for the final project.

Homework: There will be 3 problem sets, and they will be due two weeks after they were handed out via email to `erobeva@math.ubc.ca`. Late homework will not be accepted, unless there is a prior arrangement with the instructor.

If you miss a homework for a valid reason (see UBC Vancouver Senate?s Academic Concession Policy V-135), please fill out an academic concession form and bring it to the instructor. The weight of the missed homework will be transferred onto the remaining homework assignment. Note that in accordance with UBC policy for academic concessions, this form may be used ONCE per course. On a second instance, students will be expected to provide documentation.

Scribe work: Each student will also be responsible for editing and/or writing lecture notes from two lectures.

Collaboration policy: We encourage working together whenever possible. However, the handed in homework solutions should reflect each student's own understanding of the class material. It is not acceptable to copy a solution that somebody else has written.

Tentative Course Schedule

Lec.	Topic
1.	Course Overview and Motivating Examples
2.	CP decomposition - definition and properties
3.	Examples of CP decomposition - latent variable models, Gaussian mixture models
4.	Examples of CP decomposition - blind source separation, ICA, the method of moments
5.	NP-hardness
6.	Algorithms for CP decomposition - Jennrich's algorithm, alternating least squares
7.	Algorithms for CP decomposition - semidefinite relaxations
8.	Atomic norm minimization and the tensor nuclear norm
9.	Eigenvectors of tensors, the tensor power method, orthogonally decomposable tensors
10.	Overcomplete CP decompositions - the subspace power method
11.	Tensor network decompositions - motivation from quantum physics
12.	Tensor network decompositions - Tucker, MPS (aka Tensor Train), PEPS, MERA
13.	Graphical models - undirected, Markov properties
14.	Correspondence between graphical models and tensor networks
15.	Nonnegative matrix decompositions - nonnegative rank, properties, examples
16.	Nonnegative matrix decompositions - alternating least squares, EM, other algorithms
17.	Nonnegative matrix decompositions - geometric description
18.	Nonnegative tensor decompositions - properties, examples, algorithms
19.	Total positivity - properties and relationship to nonnegative tensor decomposition
20.	Graphical models - directed acyclic; Markov properties, equivalence classes
21.	Graphical models - latent variables, Markov properties
22.	Linear structural equation models (LSEM) - Gaussian vs. non-Gaussian; Darmois-Skitovich
23.	Independent Component Analysis and learning non-Gaussian LSEMs
24.	Project presentations
25.	Project presentations

References

The following references are relevant for the topics covered in this course.

1. E. Allman, J. Rhodes, B. Sturmfels, and P. Zwiernik. *Tensors of Nonnegative Rank Two.* Linear Algebra and its Applications. 473:37-53, 2015.

2. A. Anandkumar, R. Ge, D. Hsu, S. Kakade, and M. Telgarsky. *Tensor Decompositions for Learning Latent Variable Models.* Journal of Machine Learning Research, 15(80):2773-2832, 2014.

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4. S. Boyd and L. Vandenberghe. *Convex Optimization.* Cambridge university Press, 2004.

5. J. Bridgeman. *Hand-waving and Interpretive Dance: An Introductory Course on Tensor Networks.* Journal of Physics A Mathematical and Theoretical 50(22), 2016.

6. D. Cartwright and B. Sturmfels. *The Number of Eigenvalues of a Tensor.* Linear Algebra and its Applications, 432(2):942-952, 2013.

7. P. Comon and C. Jutten. *Handbook of Blind Source Separation : Independent Component Analysis and Applications.* Academic Press, Inc., 2010.

8. V. de Silva and L.-H. Lim. *Tensor rank and the ill-posedness of the best low-rank approximation problem.* SIAM J. Matrix Anal. Appl. 30(3):1084-1127, 2008.

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12. C. Hillar and L.-H. Lim. *Most Tensor Problems are NP-Hard.* Journal of the ACM, 60(6), Article 45, 2013.

13. F. L. Hitchcock. *The expression of a tensor or a polyadic as a sum of products.* J. Math. Phys. 6(1): 164-189, 1927.

14. J. Kileel and J. Pereira. *Subspace power method for symmetric tensor decomposition and generalized PCA.* Preprint: arXiv:1912.04007, 2019.

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17. L.-H. Lim. *Singular Values and Eigenvalues of Tensors: a Variational Approach.* Computational Advances in Multi-Sensor Adaptive Processing, 1st IEEE International Workshop 129-132, 2005.

18. A. Moitra. *Algorithmic Aspects of Machine Learning.* `http://people.csail.mit.edu/moitra/docs/bookex.pdf`

19. D. Mond, J. Smith, and D. van Stratten. *Stochastic factorizations, sandwiched simplices and the topology of the space of explanations.* Proceedings of the Royal Society A. 459(2039), 2003.

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