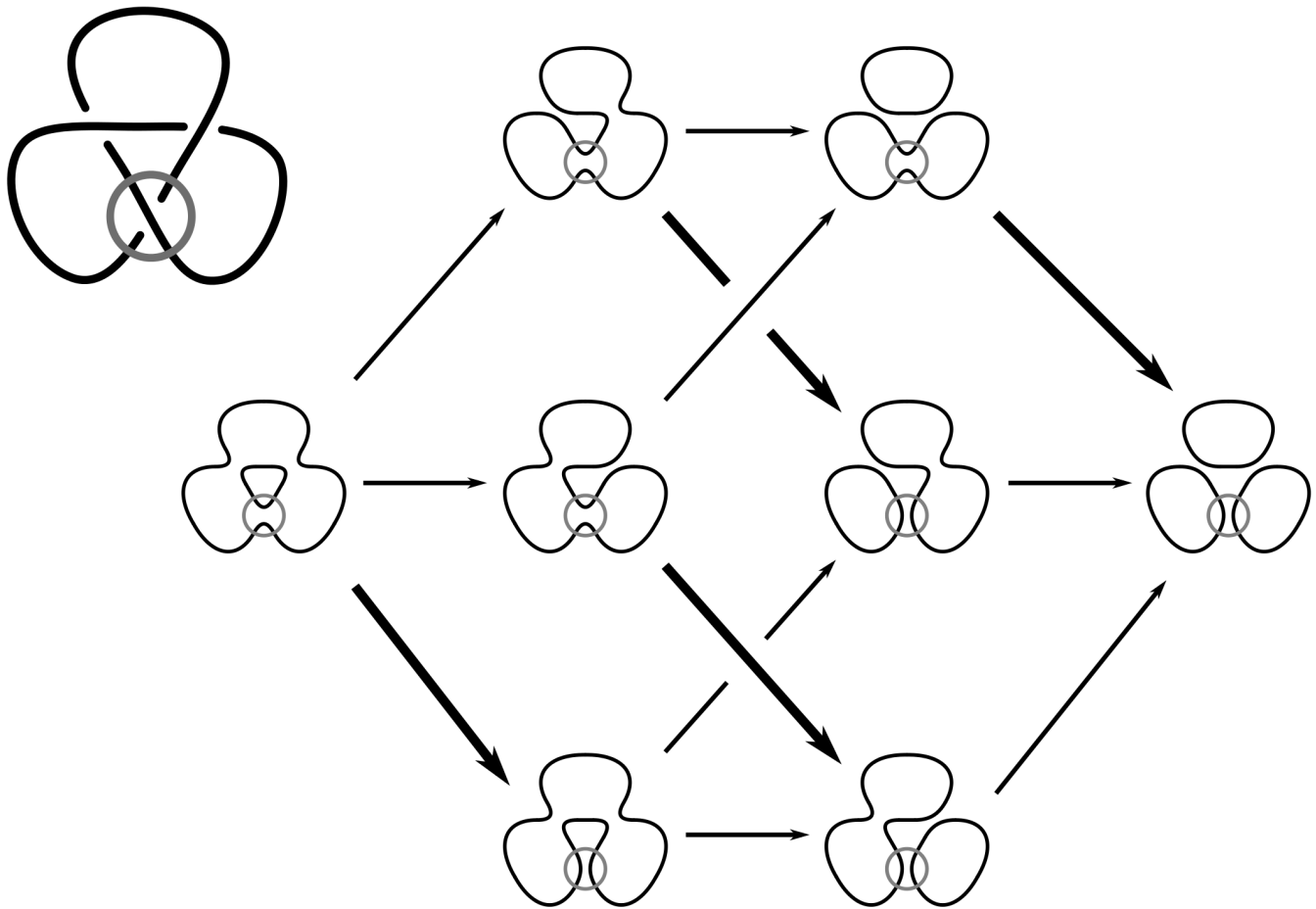




Knot homology theories

Math 603D

Fall 2020

This is a course on homological invariants of knots. This will take Khovanov homology as a central object of study, with a focus on the current state of homological invariants in low-dimensional topology, more generally, since Khovanov's initial work categorifying the Jones polynomial [Kh2000], which was posted to the arXiv just over 20 years ago.

As a starting point, this course will assume a little basic knot theory, in particular, the definition of a knot. A great reference for this is Dale Rolfsen's classic book *Knots and Links* [Ro1976].

- **Instructor**

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Office hours by appointment.

- **Where, when...**

The class meets Tuesdays and Thursdays at 1:00 pm Pacific time, using Zoom. The first lecture is on September 8, 2020.

- **Evaluation**

Your final grade will be largely based on your final presentation for the course, which will constitute an hour-long lecture on a topic related to the course material. I will work with you in order to help choose a subject that aligns with your interests; presentation topics will be settled by the end of September. You will also be required to submit an extended abstract/literature review by the end of October (worth 20% of your final grade) and submit solutions to exercises that come up in lectures (worth 10% of your final grade). This latter is essentially a participation score, though note that you must submit homework in order to receive an A⁺ in the course.

- **Resources**

I'll draw primarily on research articles in this course; references will be updated/expanded continuously (see bottom of page). There are now many survey articles and lecture notes covering various aspects of knot homology theories (generally) and Khovanov homology (specifically). For an excellent introduction to Khovanov homology see Bar-Natan's *On Khovanov's categorification of the Jones polynomial* [BN2002] and for a description of *knot homologies* that aligns closely with our course, see Rasmussen [Ra2005].

Lectures

- **Prelude: the Conway knot is not slice (September 8)**

In order to provide some context for and an overview of the course, this lecture aims to highlight the apparently crucial appeal to Khovanov homology [Kh2000]—precisely, to Rasmussen's s-invariant [Ra2010]—in Piccirillo's recent proof that the Conway knot is not slice [Pi2020]. In so doing, I'll aim to set up some context for the kinds of problems currently driving low-dimensional topology, which will be a secondary theme in the course.

For those interested in a different take, or in mathematical writing for broader audiences, there is an excellent [Quanta article reporting on Piccirillo's work](#).

- **Khovanov's construction (September 10)**

References

[BN2002] Dror Bar-Natan [On Khovanov's categorification of the Jones polynomial](#) *Algebr. Geom. Topol.* (2002)

[Kh2000] Mikhail Khovanov [A categorification of the Jones polynomial](#) *Duke Math. J.* (2000)

[Pi2020] Lisa Piccirillo [The Conway knot is not slice](#) *Ann. of Math.* (2020)

[Ra2005] Jacob Rasmussen [Knot polynomials and knot homologies](#) in *Geometry and topology of manifolds*, *Fields Inst. Commun.* (2005)

[Ra2010] Jacob Rasmussen [Khovanov homology and the slice genus](#) *Invent. Math.* (2010)

[Ro1976] Dale Rolfsen [Knots and Links](#) Publish or Perish Press (1976)