

## MATH 526: Differential Geometry II

**Course Material and Topics:** This course covers the fundamental concepts of metrics, connections, and curvature. Riemannian manifolds are smooth manifolds equipped with Riemannian metrics, which allow one to measure geometric quantities such as distance and angles and study geometric properties of curved space. This course covers core material that would be useful for many areas of mathematics and physics.

Topics to be covered:

- Riemannian metrics
- Connections
- Riemann curvature tensor, sectional curvature, Ricci curvature, scalar curvature
- Basic theory of geodesics: the exponential map, Jacobi fields, completeness
- Riemannian submanifolds, second fundamental form, and the Gauss and Codazzi equations
- Spaces of constant curvature (canonical metrics)
- Classical applications: Hopf-Rinow and Hadamard theorems, Bonnet-Myers theorem, and Synge's theorem
- Other topics, like
  - Relationships between curvature and topology, the sphere theorem
  - Comparison theorems: Rauch, Laplacian, Hessian; Toponogov's theorem
  - Topics in geometric analysis

**Prerequisites:** It will be assumed that the student has had the usual undergraduate training in analysis (for example, MATH 320), linear algebra, and basic ODE. Students should have some familiarity with differentiable manifolds (such as MATH 525 here at UBC).

**Evaluation:** The instructional format for the course will consist of lectures of 2.5 hours per week on Zoom. Homework problems will be assigned periodically. There will be no final exam.

**Textbook:** *Riemannian Manifolds: An Introduction to Curvature* by John M. Lee

[Note: The e-book can be downloaded via the UBC Library website. Click on the link above.]

### References:

- M. do Carmo, *Riemannian Geometry*
- B. O'Neill, *Semi-Riemannian Geometry With Applications to Relativity*
- S. Gallot, D. Hulin, and J. Lafontaine, *Riemannian Geometry*
- J. Cheeger, D. Ebin, *Comparison Theorems in Riemannian Geometry*
- D. Spivak, *A Comprehensive Introduction to Differential Geometry, 3rd Edition*

### Instructor:

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