UBC Math 521: Numerical Analysis of Partial Differential Equations
Winter 2020T1

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Course web page: www.math.ubc.ca/~wachs/Teaching/MATH521/math521-2020W1.html
Canvas page: https://canvas.ubc.ca/courses/62883

Lectures: Tu/Th 9:30-11:00, online

Office hours: TBA, online

General purpose:
What surface does a soap film span inside a bent wire frame? What does the electromagnetic field inside a microwave look like? How does air flow around a car? How does the shockwave of a supersonic aircraft propagate in space? How does chemotaxis work? How do rain clouds develop over the Pacific?

Partial differential equations (PDEs) model a vast range of problems from physics, chemistry, biology, engineering, meteorology, statistics, mathematical finance and many more disciplines. Virtually all real-life problems are too complex to be solved analytically and require numerical techniques such as finite differences, finite elements or finite volumes.

This course is designed to foster development of analytical, computational and professional skills. Not only will you learn how to solve PDEs numerically and how to assess the quality of your results, but you will also apply these skills in your own mini research project, you will gain effective communication skills to present your results in oral and written form and you will find out how to peer-review someone else's work.

Learning Outcomes
At the successful completion of this course, you will be able to:

(LO1) Derive characteristic features of solutions to a given PDE problem.

(LO2) Select and implement a suitable numerical method that preserves these features for first-order or second-order PDE problems.

(LO3) Describe the fundamental notions of consistency, stability and convergence of a numerical scheme.

(LO4) Calculate a priori and a posteriori error estimates for some elliptic model problems.

(LO5) Identify how the notions and techniques from the numerical analysis of PDEs are applied in fields of interest to the class.

(LO6) Follow a goal-oriented approach to written and oral forms of communication in academia, to convey scientific findings in an effective, interesting and captivating manner.

References: You do not need to purchase a textbook. There will be worksheets and notes tailored to this course. Here is a small selection of the relevant literature that you may want to have a look at:

Numerical Analysis of PDEs

1. Stig Larsson and Vidar Thomée: Partial Differential Equations with Numerical Methods. Springer, 2009.
2. Susanne Brenner and Scott Ridgway: The Mathematical Theory of Finite Element Methods. Springer, 2007.
3. Kenneth Eriksson et al: Computational Differential Equations. Cambridge University Press, 1996.
4. Philippe G Ciarlet et al: Series: Handbook of Numerical Analysis. North-Holland, 1981–2017.

Theory of PDEs

5. Lawrence C Evans: Partial Differential Equations. American Mathematical Society, 2010.

Scientific Communication

6. Michael Alley: The Craft of Scientific Presentations. Springer, 2013.
7. Ed Neal and Doug Dollar (Eds): Academic Writing: Individual and Collaborative Strategies for Success. New Forums, 2013.

Course outline/content:

1. Classification of PDEs
(a) Basic Properties
(b) Second-Order PDEs
(c) Conservation Equations

2. Second-Order Elliptic Equations
(a) Characteristic Features
(b) Finite Differences for Poisson's Equation
(c) Finite Elements for Poisson's Equation

3. Second-Order Parabolic Equations
(a) Characteristic Features
(b) Finite Elements for the Heat Equation

4. Second-Order Hyperbolic Equations
(a) Characteristic Features
(b) Finite Elements for the Wave Equation

5. Conservation Equations
(a) Characteristic Features
(b) Finite Elements for the Advection-Diffusion Equation

Grading scheme:

- 12 homework assignments (one assignment per week): 60%
- project report: 20%
- project presentation: 15%
- peer review: 5%