

SYLLABUS FOR 603D: K THEORY

1. CONTACT INFORMATION

The instructor for this course is me, Ben Williams. I may be reached at tbjw@math.ubc.ca.

The course website is <http://www.math.ubc.ca/~tbjw/KTheory/index.html>. What appears in this document is much the same as what appears on the website, and the website is more up-to-date. Please consult the website in case of any changes.

2. MEETING TIME & PLACE

The course meets MWF from 11-12 in MATX 1102. This may change if the time does not suit too many of the course attendees.

3. DESCRIPTION

This is a topics course in K -theory. The first part will deal with the elementary K -theory of rings, i.e., the classical theory of $K_0(R)$ and $K_1(R)$. We will prove the Serre–Swan theorem, relating this K_0 -theory with K -theory for topological spaces. Then K -theory as an extraordinary cohomology theory on topological spaces will be presented, and we will introduce spectra and stable homotopy theory motivated by this theory. The last part of the course will return to algebraic K -theory and discuss the problem of constructing and then calculating “higher” algebraic K -theory.

4. REFERENCES

This course is in three sections.

- For the first section, on the elementary theory, I recommend Milnor’s “Introduction to Algebraic K -theory”, [Mil71]. In this part of the course, we will also prove the Serre–Swan theorem of [Swa62].
- The second section, on the topological theory, will use [Ati18] as a partial source, although I reserve the right to substitute a different proof of Bott periodicity. We will also introduce stable homotopy using [Ada74] as a guide, although course notes will also be supplied. For different approaches to Bott periodicity, see [Kar05].
- For the third section, on higher Algebraic K -theory, the textbook source I will use is [Sri08], supported by [Ros94].

5. HOMEWORK & GRADES

Some homework assignments will be given over the course of the term. The final grade will depend wholly on these assignments.

6. OUTLINE

- (1) Elementary K -theory.
 - (a) Projective modules and vector bundles.
 - (b) Calculations of K_0
 - (c) The Serre–Swan theorem
 - (d) The definition of K_1 .
- (2) Topological K -theory
 - (a) Classifying vector bundles.
 - (b) Spectra and generalized cohomology.
 - (c) Bott periodicity.
- (d) The Chern character
- (e) Atiyah–Hirzebruch spectral sequence
- (3) Higher Algebraic K -theory
 - (a) Definition of higher algebraic K -theory.
 - (b) Dévissage
 - (c) The projective bundle formula
 - (d) K -theory as group completion
 - (e) K -theory and \mathbf{A}^1 -invariance.

REFERENCES

- [Ada74] J. F. Adams. *Stable Homotopy and Generalised Homology*. University of Chicago Press, Chicago, Ill.-London, 1974.
- [Ati18] Michael Atiyah. *K-Theory*. CRC Press, March 2018.
- [Kar05] Max Karoubi. Bott periodicity in topological, algebraic and Hermitian K -theory. In *Handbook of K-Theory. Vol. 1, 2*, pages 111–137. Springer, Berlin, 2005.
- [Mil71] John W. Milnor. *Introduction to Algebraic K-Theory*, volume no. 72 of *Annals of mathematics studies*. Princeton University Press, January 1971.
- [Ros94] Jonathan Rosenberg. *Algebraic K-Theory and Its Applications*, volume 147 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1994.
- [Sri08] V. Srinivas. *Algebraic K-Theory*. Modern Birkhäuser Classics. Birkhäuser Boston, Inc., Boston, MA, second edition, 2008.
- [Swa62] Richard G. Swan. Vector bundles and projective modules. *Transactions of the American Mathematical Society*, 105:264–277, 1962.