MATH 567 Nonlinear Wave Equations

January - April, 2019

Course Description

Wave propagation subject to nonlinear effects occurs in physical systems as diverse as the atmosphere, ocean surface waves, lasers, magnets, and quantum particles, and such systems are modelled by PDE known collectively as "nonlinear wave equations". Qualitatively, behaviour in these systems is often characterized by a competition between dispersive (spreading-out, wave-like) and nonlinear (concentrating, enhancing) effects. Mathematically, the study of these PDE has become one of the dominant research areas in analysis, combining as it does functional and harmonic analysis, and concepts from mathematical physics, Hamiltonian systems, and ODE theory. This course develops the mathematical tools used to address the most important questions concerning nonlinear wave equations as models of physical phenomena: Can solutions be uniquely defined locally in time? Do solutions exist for all time, or do they "blow up" in finite time? If a solution exists for all time, how does it look like after a long time? Does it become trivial? Does it settle down to some interesting configuration?

Topics

Here is the tentative outline. It can be adjusted according to audience background and interests.

- Mathematical background: Lebesgue integral, Lp and Sobolev spaces, Fourier transform.
- Linear wave equations: dispersion relations, solution formulas, group velocity, stationary phase, decay and Strichartz estimates.
- Nonlinear wave equations: Examples (shallow water, Boussinesq, KdV, nonlinear Schrödinger and Wave equations), symmetries, conservation laws, Hamiltonian formulation, scaling and criticality, solitary waves.
- Local in time existence and uniqueness of solutions, continuous dependence on data, Duhamel formulation, contraction mapping principle.
- Global behavior: finite time singularity, scattering theory, orbital stability of solitary waves.

References

- T. Cazenave: *Semilinear Schrödinger equations* (2003) -- full gory details of analysis of NLS
- T. Tao, *Nonlinear Dispersive Equations* (2006) -- a thorough introduction to the modern

mathematical theory of nonlinear waves. Chapters 1-3 are most relevant to us.

- G. Whitham, *Linear and Nonlinear Waves* (1974) -- a classical applied text. Part II covers many of the basic notions for linear and nonlinear dipsersive PDE.
- W. A. Strauss: *Nonlinear wave equations* (1989) -- a short, efficient overview of the mathematical state-of-the-art at the time.
- C. Sulem, P.-L. Sulem, *The Nonlinear Schroedinger Equation* (1999)
- G. Fibich, *The Nonlinear Schroedinger Equation* (2015)
- F. Linares and G. Ponce, *Introduction to Nonlinear Dispersive Equations* (2015)
- ssd: old lecture notes by myself

Files of some references will be available in a public owncloud folder, whose link will be given.

Prerequisites

Basic properties of Sobolev spaces and Fourier transform will be needed throughout the course, and will be reviewed in the beginning of the course.

Evaluation

The evaluation is based on homework assignments and class participation.

Instructor and lectures

Instructor: Dr. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, ttsai at math.ubc.ca.

Lectures: MWF, 13:00 - 13:50, Math Annex MATX 1102

Office hours: TBA, and by appointment (Tsai's schedule).