Math 551: Peturbation Methods for PDE's in Applied Mathematics

Course Material and Topics: This is a course in modern techniques in applied mathematics, focusing on perturbation methods for partial differential equations. The material provides valuable skills and resources complementary to scientific computations, mathematical modeling in applications, analysis of PDE's and dynamical systems. The general concepts and methods are illustrated and developed for a wide variety of specific problems arising in math biology, fluid mechanics, materials science, and wave propagation.

Topics to be covered:

- Singular perturbations and boundary layers for PDE's.
- Homogenization methods; Micro- vs. Macro-scale, Wave propagation through rough boundaries, diffusion in layered media. Deriving "effective equations" from averaging over microscale behavior in biology.
- Pattern Formation Problems: Turing stability analysis, amplitude and envelope equations, dynamics of fronts and interfaces with applications to math biology and materials science.
- Strong localized perturbation theory; Localization problems for eigenvalues, diffusion, mean first passage time, fluid flow, coarsening problems in materials science, quorum sensing behavior in biology.
- WKB for partial differential equations; high frequency waves, eigenvalue asymptotics.
- Singularities for PDE's; blow-up and focusing behavior, self-similarity.
- Steepest descent methods, Stokes phenomena, and applications.

Prerequisites: Students should have a working knowledge of Applied PDE (such as MATH 400) and of basic Complex Variables (M300). Some experience with mathematical modeling in either continuum mechanics or biology is also desirable.

Evaluation: The instructional format for the course will consist of lectures of 3 hours per week. There will be 6 homework assignments. A final written project (20 page maximum), based on readings of a current research topic, will be due in April. Students are to make a short 15 minute in class presentation of their project in April. There is no final exam. The grading is 60% homework and 40% project.

Possible references: I will provide an online set of course notes for the various topics, as there is no one book that covers the core topics. The following books are relevant to the course.

- J. Kevorkian and J. Cole; Multiple Scale and Singular Perturbation Methods, Springer, (1996). Mathematics.
- M. Holmes; Introduction to Perturbation Methods, Springer, (1995).
- R. Johnson; Singular Perturbation Theory; Mathematical and Analytical Techniques with Applications to Engineering, Springer, (2005).
- P. Grindrod; Theory and Applications of Reaction-Diffusion Equations, Clarendon Press, (1996).