MATH 548: Discrete probability

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Class hours: Tue-Thu. 14:00–14:30, MATX 1118.
Office hours: after class, or by appointment.
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Course website: `www.math.ubc.ca/~angel/548`

Course Outline

We will discuss some of problems and modern tools of discrete probability. Topics will be some of the following:

1. Random walks: their relation with electrical networks, Green's function, harmonic potential.
2. Analysis of Markov chains: Spectral theory, mixing times isoperimetric estimates, Markov chain Monte-Carlo simulations.
3. Random graphs and percolation: Erdos-Renyi random graphs, random regular graphs, Galton-Watson trees, preferential attachment and small world phenomena.
4. Ergodic theory: Birkhoff's and mean ergodic theorems, recurrence theorems, applications (van-der Waerden's and Szemeredi's theorems).
5. Percolation: Russo-Seymour-Welsh technology, the Harris-Kesten theorem, critical and off-critical behaviour, conformal invariance (Smirnov's theorem).
6. Introduction to statistical mechanical models: The Ising, Potts, hard core models and dimers.

Prerequisites

You should be familiar with basic probability and Markov chains. For maximal benefit, students should have some knowledge of measure theory, and Martingales.

Evaluation

The grade will be based on assignments throughout term, normally due one to two weeks after the relevant topics are covered in class.

- You are encouraged to collaborate in solving the problems, and should indicate on your assignments who you collaborated with. However, each student must write their solutions independently. You may share ideas but you may not share your written work.
- Late assignments will not be accepted for credit.

Textbooks and References

There is no single book that covers all parts of the course well. The following cover various aspects of the course, and can be used as references.

- R. Lyons, Y. Peres. Probability on Trees and Networks. CUP 2016.
- D.A. Levin, Y. Peres, E.L. Wilmer. Markov chains and mixing times. AMS 2009.
- F. Spitzer. Principles of random walk, 2nd ed., Springer 1976.
- G. Lawler, V. Limic. Random Walk: A Modern Introduction.
- G. Grimmett. Percolation, 2nd ed., Springer 1999.
- B. Bollobás, O. Riordan. Percolation. CUP 2006.
- B. Bollobás. Random graphs. CUP 2001.
- D. Williams. Probability with Martingales. CUP 1991.