Math 615

Quantum Invariants of Calabi-Yau threefolds

2016/2017 Term 1

Instructor: Jim Bryan

Time: TBA

Place: Math Room TBA

Course Description:

In this course I will discuss quantum invariants of Calabi-Yau threefolds. "Quantum invariants" in this context is a catch-all phrase referring to deformation invariants constructed in algebraic geometry which are mathematical analogs of quantities arising in string theory. The invariants we will consider are Gromov-Witten invariants, Donaldson-Thomas invariants, Pandharipande-Thomas invariants, and Gopakumar-Vafa invariants. They can all be regarded as theories which provide virtual counts of curves on a Calabi-Yau threefold. We will study the structure which underlies these invariants and the various relationships (many of which are conjectural) between the invariants as well as techniques for computing these invariants.

Course Outline:

1. Introduction. Calabi-Yau manifolds.
2. Invariants from virtual curve counting
    - Gromov-Witten invariants
    - Donaldson-Thomas invariants
    - the GW/DT correspondence
    - Pandharipande-Thomas theory
    - the PT/DT correspondence
    - Gopakumar-Vafa theory
    - The Gopakumar-Vafa conjecture
3. Motivic DT theory
    - Motivic stack functions, the Hall algebra of a CY category
    - Motivic Milnor fiber and the motivic Thom-Sabastiani theorem.
    - Motivic DT invariants a la Kontsevich-Soibelman
4. Computations
    - Localization and the topological vertex
    - Hall algebra proof of the PT/DT correspondence

- Motivic Donaldson-Thomas invariants of 0-dimensional sheaves
- Power structures on motives
- DT/PT invariants of local curves
- Elliptically fibered CY3 and combining motivic/localization methods

Reference:

A good overview is the paper 13/2 ways to count curves by Pandharipande and Thomas. The references within this paper provide an excellent guide to the literature.

Homework:

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