Math 605D Topics in Applied Math: Appl. & Comput. Harmonic Analysis

Instructor: Ozgur Yilmaz

First meeting: Tuesday, Jan 5, 9:30 am in BUCH B216

Course Description: In this course, we will cover some "classical" results in applied and computational harmonic analysis that include classical sampling theory, frame theory, short-time Fourier transform and Gabor expansions, and wavelets. We will also tie all these to various applications including transform coding and processing of various classes of signals such as natural images, audio, and seismic data and images (including sparse approximation and compressed sensing). The course will be self-contained, though background in functional and harmonic analysis as well as signal processing would be useful.

Text: The course will not follow any textbook, though we will use the following as supplementary text:

- An introduction to frames and Riesz bases, O. Christensen
- Harmonic Analysis and Applications, J. J. Benedetto
- Ten Lectures on Wavelets, I. Daubechies
- A Basis Theory Primer, C. Heil
- A Wavelet Tour of Signal Processing, S. Mallat
- Foundations of Signal Processing, M. Vetterli, J. Kovacevic, and V. Goyal

A detailed outline is provided below (1 lecture=80 minutes):

1. Motivation: What is "applied and computational harmonic analysis" good for? *(1 lecture)*
    - Signal acquisition: from "analog" to "digital": sampling and quantization
    - Processing and storage: denoising and transform coding
    - lmportant decompositions: Fourier, short-time Fourier (Gabor), wavelet, ...
2. Mathematical Preliminaries *(2 lectures)*
    - A need-based overview of Banach and Hilbert space theory
3. Fourier series *(1.5 lectures)*
    - L^2 theory
    - Brief overview of L^1 theory and pointwise convergence
    - Examples
4. Fourier transform on the line *(1.5 lectures)*
    - L^1 theory
    - Basic properties and examples
    - Extension to L^2
    - Relationship between Fourier series and Fourier transform

5. Sampling theory *(2 lectures)*
    - Classical sampling theorem: two proofs
    - Oversampling: quantization and amplitude errors
    - Undersampling: aliasing
    - Irregular sampling: statement of the problem
6. Frame theory *(2 lectures)*
    - Riesz bases and application to irregular sampling
    - Frames: basic definitions, properties, examples
7. Finite dimensional world *(2 lectures)*
    - DFT and its fast implementation (i.e.,FFT)
    - Discrete cosine transform (DCT)
8. Short-time Fourier transform (STFT) *(3 lectures)*
    - Continuous short-time Fourier transform
    - Discretization: Gabor frames
    - Time-frequency localization
    - STFT in finite dimension
9. Wavelet transform *(4 lectures)*
    - Continuous wavelet transform
    - Discretization: wavelet frames
    - Multiresolution analysis: construction of orthonormal wavelet bases
    - Discrete wavelet transform
10. Applications *(3 lectures)*
    - Transform coding and compression
    - Examples from practice: compression of images using wavelets, blind separation of speech signals using STFT,. . .
    - Sparse approximation and compressed sensing