MATH 421/510 201 2020W Real Analysis II

Section 201: MWF 11:00-11:50. Credit value: 3 credits. Please note that the SSC says, incorrectly, that this will be an in-person class. The course will be entirely online.

Instructor: Professor I. Laba

- **Bio:** Ph.D. 1994 (University of Toronto). At UBC since 2000. Full Professor since 2005.
- **Email:** ilaba@math.ubc.ca
- **Office hours:** Friday 11-11:50, with additional times TBA.
- If you cannot attend regular office hours due to schedule conflict, you can request an appointment. Please make your request at least one day in advance. Our schedules can fill up, so that drop-ins and same-day requests for appointments can be difficult or impossible to accommodate.
- The best way to contact me is by email. Please note that email received on evenings and weekends will be answered on the next business day. (If you are not sure how to email a professor, see here for templates and recommendations. [http://www.math.ubc.ca/~ilaba/teaching/email.html](http://www.math.ubc.ca/~ilaba/teaching/email.html))

**Prerequisites:** MATH 420/507, or equivalent background in measure theory and real analysis.

**Course-level learning objectives:**

- Learn the basic concepts of functional analysis, including normed vector spaces and Banach space, linear operators and linear functionals, operator spaces and their fundamental theorems, Hilbert spaces, applications to Fourier analysis.
- Explore the geometry of infinite-dimensional spaces.
- Practice formal mathematical proofs and high quality mathematical writing.

**Recommended learning materials:**

- **Supplementary (optional) textbooks:**
• **Other resources:** there are many texts and monographs on functional analysis, e.g. Reed and Simon, Yosida, Dunford and Schwartz. Some books, especially the older ones, may use different notation or conventions.

• **Homework assignments** (will be posted on Canvas)

**Course topics and tentative schedule** (chapter and section numbers refer to Folland unless stated otherwise):

- Normed linear spaces, Banach spaces, linear operators, boundedness and continuity, operator norm, linear functionals and dual spaces (Sections 5.1 and part of 5.2; 2 weeks, tentatively Jan. 11-22)
- Applications to function spaces (Sections 6.1 and 6.2; 1-2 weeks, tentatively Jan. 25 - Feb. 1)
- More on linear functionals and dual spaces: convexity, the Hahn-Banach theorem, hyperplane separation (Section 5.2; 1-2 weeks, tentatively Feb. 3-12)
- Linear operators, adjoints, range and kernel, strong, weak and weak* convergence, with applications to function spaces (Section 5.4; for an additional reference, try Lax, Chapters 10 and 11. 2 weeks, tentatively Feb. 22-March 5)
- Principle of Uniform Boundedness for linear functionals, with applications (Conway, Chapter III, Section 14, or Folland, Section 5.3. Both Conway and Folland state a more general version, for operators instead of functionals. 1-2 weeks, tentatively March 8-15)
- Weak sequential compactness, Baire Category Theorem, Open Mapping and Closed Graph Theorems (Section 5.3; 1-2 weeks, tentatively March 17-26)
- Hilbert spaces (Section 5.5; 2-3 weeks, tentatively March 29 - April 14).
- If we have time at the end of the course, we will finish with (a special case of) the Riesz Representation Theorem: the dual of C(X), where X is a compact subset of R^n.

A more detailed tentative schedule of class topics and textbook sections covered each week will be posted here and updated on a regular basis.

**Course structure and learning activities:**

- **Textbook:** *Real Analysis* by Folland will be the main reference, and we will also use the supplementary textbooks as needed. Folland and Conway are a little bit terse; Lax offers a more intuitive approach and PDE applications, but can be less rigorous at times.

- **Lectures and Q&A, Mondays and Wednesdays 11-11:50, Zoom:** the lectures will provide an introduction to the material (the main points, frequently asked questions, etc). Questions in class are encouraged and I will try to address them as we go. The lecture and Q&A will be recorded and available for asynchronous viewing after class. I will try to upload the class outline to Canvas in advance.

- **Discussion and office hours, Fridays 11-11:50, Zoom:** this will be a place to discuss homework, ask questions, etc. **All course-related questions, not necessarily just about the**
current homework, are welcome.

- **Discussion boards:** Both Piazza and Discussions on Canvas will be available.
- **Homework** will be assigned biweekly. Each assignment will have only 2-4 questions, but that will include proofs, and you will have to include complete, correct, and clearly written explanations and justifications for your work. You will be graded both on the correctness of your mathematics and on the quality of your mathematical writing. The correct answer alone will **not** be sufficient.
- **Verification by Videoconferencing:** From time to time, I may ask selected students to explain their homework solutions to me, either live or via a recording, with the webcam on.
- **Discussion boards:** Both Piazza and Discussions on Canvas will be available.
- **Additional practice:** Suggested textbook problems will be posted as needed. Moreover, in past editions of Math 421/510, some (not all) students reported that they benefitted from a review of linear algebra (e.g. reviewing matrices and matrix operations before studying linear operators). From time to time, I may post WebWork review assignments on linear algebra material for this purpose. Those will be **strictly optional** (not for credit).

**Suggested learning strategies:** My goal is to try to meet you where you are and be flexible with the learning options available to you. Some possible ways to plan and organize your work are as follows.

- **If you enjoy live group work and videoconferencing:** Attend the online live sessions. Try to read the corresponding textbook section in advance, so that you are ready to ask questions during the session. Work on your assignments in groups, either on the discussion boards or via videoconferencing (the Course Room on Collaborate Ultra is always open by default).
- **If you prefer working with written materials and/or have low bandwidth:** Work through the selected textbook sections (I strongly recommend that you should have the Folland textbook, so that it will be easier to be on the same page), participate in discussion boards, work on homework assignments on your own or in a Piazza chat group, check the posted solution sets afterwards, check with me and/or the TA if you have questions.
- **If you cannot participate synchronously (e.g. time zone conflict) but would still like to take advantage of video resources and group work where possible:** Watch the recorded lectures and Q&A. If there is a question you would like me to answer on video, you can post it on Canvas or email it to me in advance. Otherwise, work with written materials as described above.

**Your course mark** will be based on homework assignments (95%) and two engagement surveys (5%). The grades may be scaled at the end of the course. There will be no final exam.

- **Homework assignments:** Each assignment will be posted at least a week in advance. Your solutions are to be uploaded to Canvas and will be graded online. Late assignments will not be
accepted. To allow for minor illnesses and other emergencies, the lowest homework score will be dropped with no questions asked.

- **Engagement surveys:** twice during the semester, tentatively in early February and again in March, I will ask you to answer 2-3 questions about your engagement in the course. I will use the results to fine-tune the course structure, and may follow up with you as needed. These will be worth 5% of your grade (2.5% for completing each one).

**Academic concession:** The rules and procedures for obtaining academic concession are governed by [UBC Policy V-135 on Academic Concession](http://www.calendar.ubc.ca/vancouver/index.cfm?tree=3,329,0,0). The details in this course are as follows.

- **Late or missed homework:** Late assignments will not be accepted, in order to discourage pile-ups of overdue workload, keep the TA's work schedule consistent, and allow the timely posting of solution sets for everyone. (If you cannot complete an assignment before the deadline for a valid reason, see below.)

- **To account for minor illnesses and emergencies,** the grading policy allows for one homework assignment to be missed with no penalty. Academic concession requests involving two or more missed assignments should be accompanied by the [Department of Mathematics Academic Concession self-declaration form](http://www.math.ubc.ca/Ugrad/ugradForm/Student_Declaration_Academic_Concession_MATH.pdf) and submitted as soon as reasonably possible. The Academic Concession form can be used for medical issues involving you or your family members, as well as other circumstances such as a "challenging emergency/unanticipated situation". I will be interpreting this liberally, and will only require a general description of the situation without personal detail (for example, you can say that you were sick without providing medical details). The usual remedy will be to have your course grade based on your remaining work.

- Please note that academic concession for certain reasons, such as valid schedule conflicts that can be foreseen, must be requested in advance and may require additional documentation.

**Academic misconduct:** UBC takes cheating incidents very seriously. After due investigation, students found guilty of cheating on tests and examinations are usually given a final grade of 0 in the course and suspended from UBC for one year. See here for more information. ([http://www.calendar.ubc.ca/vancouver/?tree=3,54,111,959](http://www.calendar.ubc.ca/vancouver/?tree=3,54,111,959))

- While students are encouraged to study together, you should be aware that blatant copying of another student's work is a serious breach of academic integrity. Your final write-up should be your own.

- Academic misconduct includes misrepresenting a medical excuse or other personal situation for the purposes of postponing an examination or quiz or otherwise obtaining an academic concession.
Additional help:

- **Office hours:** I'm making Friday's class time available for discussion and office hours. If there is demand for additional office hour time, I will schedule and announce it.

- **Individual videoconferencing consultations by appointment** are available, on a limited basis, during normal business hours (M-F 9-5, Vancouver time). Please make your request at least one day in advance. Our schedules can fill up, so that drop-ins and same-day requests for appointments can be difficult or impossible to accommodate. Please also let me know in advance what you would like to discuss (e.g. the general nature of your inquiry, the homework or textbook question you'd like to talk about) so that I can look it up before the appointment and we can use the videoconferencing time efficiently.

**Statement about the University's values and policies, mandated by UBC Policy V-130 (http://www.calendar.ubc.ca/vancouver/index.cfm?tree=3.328.0.0):** UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious, spiritual and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available [here](http://senate.ubc.ca/policies-resources-support-student-success).

### Course Summary:

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