1. [15]
The flow field given by a source located at \( z = 1 \) is modified by the introduction of an infinite barrier at \( x = 0 \). For what values of \( y \) is the speed on the barrier \( k \) times the speed at the same location without the barrier? What is the possible range of \( k \)? Explain.

2. [30]
(a) Evaluate:
\[
I = \int_{0}^{\infty} \frac{xdx}{8 + x^3}.
\]
Hint: you should consider using a contour that includes the ray \( \theta = \frac{2\pi}{3} \).

(b) By integrating around the finite branch cut \([-1, 1]\) (and using symmetry), evaluate
\[
J = \int_{0}^{1} \frac{x^4dx}{(1 - x^2)^{\frac{1}{2}}(1 + x^2)}.
\]

(c) Show by considering the two cases \( x > 0 \) and \( x < 0 \) that
\[
p.v. \int_{-\infty}^{\infty} \frac{e^{ix\omega}}{\omega^2 - 1}d\omega = -\pi \sin |x|.
\]

3. [20]
(a) Find a conformal mapping \( w = f(z) \) that takes the region \( \{|z - 1| < \sqrt{2}\} \cap \{|z + 1| < \sqrt{2}\} \) into a portion of the right half plane.
Draw rough sketches of the regions in both the \( z \) and \( w \) planes.
It might be useful to check the image of \( z = 0 \).

(b) Find \( \phi(x, y) \) that satisfies
\[
\nabla^2 \phi = 0 \text{ in } \{|z - 1| < \sqrt{2}\} \cap \{|z + 1| < \sqrt{2}\}
\]
with: \( \phi = 1 \) on \( |z + 1| = \sqrt{2} \), and \( \phi = 2 \) on \( |z - 1| = \sqrt{2} \).

4. [20]
Let \( f(x) \) and \( g(x) \) be two absolutely integrable functions. Solve the boundary-value problem using Fourier transform, assuming \( |u(x, y)| \) decays rapidly as \( (x, y) \to \infty \).
\[
u_{xx} + u_{yy} = f(x)e^{-y}, \quad -\infty < x < \infty, \quad 0 < y,
\]
\[
u(x, 0) = g(x), \quad -\infty < x < \infty.
\]

5. [15]
Solve the following ODE using Laplace transform and Bromwich formula:
\[
y''' + y = 1, \quad (t > 0); \quad y(0) = y'(0) = 0, \quad y''(0) = 1.
\]
Do not replace exponential functions by trigonometric functions in your solution.