1. Consider the differential equation \( \frac{dy}{dx} = 2x - y \). (*)

(a) Find the general solution of (*).
(b) Find a particular solution (is there only one?) of (*) satisfying the initial condition \( y(0) = -2 \).
(c) Sketch the solution curves of (*).

2. Find the inverse function \( f(t) \) of the Laplace transform for

(a) \( F(s) = \frac{s}{s^2 - s - 2} \);
(b) \( F(s) = \frac{1}{s^2} \left[ e^{-t} - (s^2 + s)e^{-2t} \right] \).

In each case, evaluate \( f(3) \).

3. Solve the initial value problem: \( y'' + 4y = \cos t, \quad y(0) = y'(0) = 0 \).

4. Solve the following initial value problem for \( t > 0 \) and sketch its solution:

\( y'' - y' = \delta(t - \pi), \quad y(0) = 1, \quad y'(0) = 0 \).

[Note that \( \delta(t) \) is the Dirac delta function.]

5. Find the general solution of each of the following differential equations:

(a) \( y'' + 2y' + y = 0 \);
(b) \( \frac{d^4y}{dx^4} + 4 \frac{d^2y}{dx^2} = 0 \).

6. (a) Solve the initial value problem

\[
\begin{align*}
\frac{dx}{dt} &= x - 3y, \\
\frac{dy}{dt} &= 3x + 7y,
\end{align*}
\]

with \( x(0) = 0, y(0) = 1 \).

(b) Sketch the trajectory of the solution of (a) in the \( xy \)-phase plane for \(-\infty < t < \infty \), indicating by arrows the direction of increasing \( t \).

(c) Solve the initial value problem

\[
\begin{align*}
\frac{dx}{dt} &= x - 3y + 1, \\
\frac{dy}{dt} &= 3x + 7y + 1,
\end{align*}
\]

with \( x(0) = 0, y(0) = 1 \).
(20) 7. Consider the system
\[
\begin{align*}
\frac{dx}{dt} &= -y(y - 2), \\
\frac{dy}{dt} &= (x - 2)(y - 2),
\end{align*}
\]
for \( t > 0 \).
(a) Sketch the \( y(t) \) component of the solution of this system for each of the following two sets of initial conditions:
(i) \( x(0) = 1, \ y(0) = 2 \);
(ii) \( x(0) = 1, \ y(0) = 3 \).
(b) Suppose one has the initial condition \( x(0) = \alpha, \ y(0) = \beta \). Find all values of \( \alpha \) and \( \beta \) for which
\[
\lim_{t \to \infty} x(t) = A \quad \text{and} \quad \lim_{t \to \infty} y(t) = B
\]
both hold for some constants \( A \) and \( B \).
(c) Find \( A \) and \( B \).
# TABLE OF INFORMATION

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